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MANUAL OF METEOROLOGY

VOLUME III

THE PHYSICAL PROCESSES OF WEATHER

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To

SIR J. J. THOMSON, O.M.

Master of Trinity

SIR RICHARD T. GLAZEBROOK, K.C.B.

Fellow of Trinity

These recollections, reflexions and refractions of the youthfulness of the Cavendish Laboratory are affectionately inscribed

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PREFACE

THE Preface to Part IV which introduced this Manual to the reader in 1919 contemplated as a preliminary a historical introduction and a statement of the general meteorological problem at the present day, to be followed by Part I "a general survey of the globe and its atmosphere," Part II "the physical properties of air," and Part III the setting out of "the dynamical and thermal principles upon which theoretical meteorology depends and which find their application in Part IV." It was further contemplated that Parts II and III might be included in a single volume.

The historical introduction claims its place as Vol. I, and the general survey of the globe and its atmosphere as Vol. II.

The endeavour to represent the debt which meteorology owes to the achievements of experimental physics has resulted in an alteration of the plan. The thermal principles operative in the atmosphere were found to be an essential part of the study of the physical properties of air. And the mode of treatment led automatically to the consideration and then to the reconsideration of the customary meteorological methods of dealing with the reaction of the atmosphere to the thermal treatment which it receives in the natural course.

The reconsideration opened out upon some suggestions for the use of entropy as a meteorological element in various ways that invited exploration. In particular it has been found possible to regard an *isentropic surface* as a practical alternative for sea-level or some other *horizontal surface* on which to place the facts about weather. Only the beginnings of the exploration have been made and it is hoped to enlist the reader's assistance in its prosecution.

To break off that exploration in order to include the recital of the achievements of Newtonian dynamics in the domain of meteorology would be a change of key-note more suitable for another volume, to include what has already been printed in Part IV, than another chapter which would leave Part IV as a detached appendix. The new volume is the more natural since the original Part IV is already out of print.

So the portion which was to be the physical properties of air appears as a separate volume, III, with the more descriptive title *The Physical Processes of Weather*, and the statement of the dynamical principles on which theoretical meteorology depends is postponed to form, with Part IV, a prospective Volume IV with the title *Meteorological Calculus, Pressure and Wind*.

Apart from obligations noted on page 9 to authors and collaborators for help in the substance of the work and the mode of its arrangement a number of acknowledgments are here gratefully recorded:

to the Meteorological Office, Air Ministry, for the continued assistance of Miss Elaine Austin, and for access to many original documents and publications;

to H.M. Stationery Office for permission to reproduce Fig. 26 from the *Meteorological Magazine*, and Fig. 32 from the *Marine Observer*, and a number of extracts from the publications of the Meteorological Office: the source of each is duly noted in the text;

to the Royal Society for permission to reproduce illustrations and text of Professor C. T. R. Wilson, Dr G. C. Simpson and Dr G. M. B. Dobson; to the Royal Meteorological Society for the loan of the block of Fig. 120, and of the original photographs for Fig. 123; to Macmillan and Co. Ltd., for permission to reproduce from *Nature* Fig. 30, and numerous extracts from the *Dictionary of Applied Physics* and other sources which are noted in the text; to Longmans, Green and Co. Ltd., for the extracts from Maxwell's *Heat*, from Campbell's *History of the Cavendish Laboratory*, and from John Tyndall's works; and to Mrs Tyndall for her concurrence and for suggestions in connexion therewith; to the author and publishers of McAdie's *Principles of Aerography*, for permission to quote the extract on Ice-storms, peculiarly applicable to the navigation of airships; to E. Stanford and Co. Ltd., for the outline of the map on Mollweide's projection for Chapter X;

to the Cambridge University Press, its manager, printers and readers, for the comely appearance of the book.

NAPIER SHAW

31 *October* 1929

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SYMBOLS

Unalterable constants: $\pi = 3{\cdot}14159$, $e = 2{\cdot}71828$, $\log_{10} e = 0{\cdot}43429$, $\log_e 10 = 2{\cdot}30258$.

For the best of reasons efforts have been made to systematise and arrange a notation for the symbols which are required for the multitude of quantities employed in the analysis of physical processes and the mathematical operations which those quantities may have to undergo. The efforts which we have in mind at the moment are those of the International Commission on the Unification of Physico-Chemical Symbols extended by a special committee of the Physical Society of London[1], McAdie's list of symbols to secure uniformity in aerographic notation[2], and the array of symbols employed by L. F. Richardson[3].

H. Jeffreys has called attention to diversity of practice with reference to latitude and longitude[4].

The material available for a system of symbols consists of the 26 letters of the Latin alphabet, 24 letters of the Greek alphabet with some traditional mathematical signs[5]. The Gothic alphabet is also available but is little used in manuscript, still less in typescript. Some additional symbols from other alphabets are used by Richardson.

We may remind the reader that one phase of the problem which this list of symbols suggests has been solved for meteorological observations on the analogy of the traditional mathematical signs, by international agreement upon a list of symbols for weather, international hieroglyphics, specially devised for recognisable writing as set out in chapter II of volume I. We have tried to make use of the idea in the symbols for distinguishing the energy of long-wave radiation from that of short-wave radiation on p. 164 and the representation of radiation by opposing arrows on p. 200.

The 26 letters of the Latin alphabet, judiciously used, can supply 104 symbols, capitals and small letters, roman and italic; and the Greek alphabet 35 symbols, thirteen of the upper case letters are identical with the Latin capitals.

Besides the author, who writes or types, there is also the printer to be considered. He has traditions for his guide in the selection of type as between uncial and cursive, roman and italic. A writer often leaves the printer to his discretion and draws no distinction himself between u.c. and l.c., rom. or ital. Some understanding is accordingly required.

So far as our observation goes many writers on physical subjects draw no appreciation of difference between the six classes of type; but, with care, one can detect an inclination for the use of:

(i) lower case italic letters for algebraical variables such as x, y, z, t or for unknown constants, a, b, c;

(ii) lower case or upper case roman for numerical constants of known value, h, c.

These two almost imply

(iii) lower case, or upper case, roman for symbols of algebraical operation;

(iv) lower case or upper case roman to denote the units in which a quantity is expressed numerically; one of the two should suffice, namely lower case g, m, h.

[1] *Physical Society Proceedings*, vol. XXVI, 1913–14, p. 181; vol. XXVII, 1914–15, p. 105.

[2] *Annals of the Astronomical Observatory of Harvard College*, vol. LXXXIII, pt. 4, Cambridge, Mass., 1920, p. 169.

[3] *Weather Prediction by Numerical Process*, Camb. Univ. Press, 1922, pp. 224–7.

[4] *Q. J. Roy. Meteor. Soc.* vol. XLVIII, 1922, p. 30.

[5] 'Mathematical notation through the centuries,' T. L. H., *Nature*, vol. CXXIV, 1929, p. 4.

The resolutions of the Committee of the Physical Society include

TYPOGRAPHICAL

Capitals and small letters. For electrical quantities varying harmonically, capitals should stand for the amplitude and small letters for the value at any instant.

Greek letters. Where possible, Greek letters should be used for angles and for specific quantities.

Subscripts. The use of subscripts for components of vectors should be discouraged. As a general rule subscripts should be avoided.

Abbreviations for names of units. Ordinary type should be used for the symbols of units, and not clarendon.

In agreement with the recommendations of the International Electrotechnical Commission, the International Commission on the Unification of Physico-Chemical Symbols, and other bodies, the Committee recommend:

That italic, not roman, letters be used as symbols for the magnitudes of quantities in all branches of physics. This applies to capitals as well as to lower case letters.

To these four let us add:

(v) A symbol should be regarded by the printer as a symbol and not the abbreviation of a word. It does not require to be followed by a ., but may have its comma like an ordinary substantive when there is a succession.

(vi) There seems no good reason why a double letter or a syllable should not be employed instead of borrowing a symbol already in use, especially is this the case with regard to (iv). There seems no reason for example why mu should not be used to indicate micron instead of employing the much used symbol μ, just as sec is used for a second of time.

We have found the use of *tt* and *hh* quite convenient, we have contemplated saving symbols by using *ii* for angle of incidence, *rr* for angle of refraction, and *nn* for index of refraction.

(vii) Traces of system are apparent in the notation for fluxions $\dot{x}$, $\ddot{x}$, and for such related quantities as for a mean value and the departures therefrom $\bar{x}$, x', or for the original position of a particle affected by a wave and its displacement from that position x, ξ.

(viii) Accents (except those used to denote minutes and seconds of angle) and suffixes are at the discretion of the writer and printer.

In the symbolism of this book we have endeavoured (not always successfully) to keep these eight guiding principles in mind.

In order not to interrupt a train of thought it is natural for a writer on any occasion to use the first symbol that the point of his pen happens to form, guided by some reminiscence of previous habit or by the subconsciousness that a new idea should not claim a new symbol until it has established its respectability. Needless to say, a reader who is concerned with a writer's single paragraph views the matter from a different point of view; much loss of time (among other things) is involved in the uncertainty as to what a writer might actually mean by symbols of dual or triple significance. Knowledge of the practice of others is a step towards organisation. We have accordingly tabulated the usage that is to be found either in this volume or other volumes bearing on the same subjects and present the result here. The list is by no means complete but it may be helpful as indicating the symbols that a "compleat" meteorologist may meet with in the course of his reading.

Authority for the quotation of the meaning of the several symbols is indicated as follows:

(0) this Manual; (1) other meteorological books; (1*) McAdie or Richardson, (2) ancient physics including thermodynamics and optics; (2*) recommendations

of the Committee of the Physical Society; (3) modern physics, electricity, radiation, electron theory; (4) astronomy; (5) dynamics and hydrodynamics; (6) statistics and algebra; (7) aeronautics.

The abbreviation *var* means that the symbol is found as a *variable* in an algebraical equation.

TABLE OF SYMBOLS USED IN THIS VOLUME

Arranged according to the alphabets employed, namely: upper case roman, lower case roman, upper case italic, lower case italic, greek uncial, and greek minuscule.

UPPER CASE ROMAN

A Absorption band (2); absolute temperature (1); Ångström unit (*passim*); heat-equivalent of work (1*); ampere (2*).

Å Ångström (1*).

B Absorption band (2); constant of integration in expressions for intrinsic energy and total heat of vapour (2); magnetic induction (2*).

C Constant (1*); electric capacity (2*); coulomb (2*).

D Total differential.

E East (*passim*); energy (1*); voltage (2*).

F Magnetic flux (2*); farad (2*).

G Gramme in C.G.S.; conductance (2*).

H Height of homogeneous atmosphere (0); magnetic force (2*); henry (2*).

I Electric current (2*); intensity of magnetisation (2*).

J Mechanical equivalent of heat (2); joule (2*).

K Turbulence transmission of heat and motion (1*); electric capacity (2*).

L Latent heat (0, 2); self-inductance (2*); west longitude (4).

M Magnetic moment (2*); mutual inductance (2*).

N North (*passim*); number of atoms or molecules (2); Avogadro's constant (1*).

O Radius of earth (0).

P Electric polarisation (2*); power (2*).

Q Heat-energy (1*); electric charge (2*).

R Gas-constant (0, 1*); electric resistance (2*).

S South (*passim*); second in C.G.S.; entropy (1*).

T Temperature, absolute or tercentesimal (0, 2) (with suffix to denote the scale 1*); period (2*).

U Velocity of sound (2); internal energy (1*).

V Speed of waves (0); voltage (2*); volt (2*).

W West (*passim*); energy and work, see *w* (2*); watt (2*); external work (1*).

X Absorption band (2); cross-section of pipe or nozzle (2); reactance (2*).

Y Absorption band (2).

Z Absorption band (2); impedance (2*).

LOWER CASE ITALIC

a A constant or coefficient (0, 6), amplitude in a Fourier series (0, 6), specific weight of air at any point at any moment (7), radius of the earth (1*), constant in van der Waals formula $(V-b)(P+a/V^2)=RT$ (2*), specific gravity of water vapour referred to dry air (1)

b A constant (0, 6), gas-constant (1*), co-volume in van der Waals equation (2*), decay coefficient (2*).

bb Pressure-gradient, *var* (Buys Ballot) (0)

c Constant (0, 6), eddy-viscosity (1*), co-aggregation volume equals *a*/*RT* in approximate formula $V = RT/P + b - c$ (2*)

c Preferably c, specific heats of air (0, 1, 2), velocity of light (3, 4)

d Deviation, *var* (0, 6), partial pressure of dry air (0), density of water vapour (0), density of gaseous air (0), vertical distance (1)

d Preferably d, symbol of differentiation (1*, *passim*)

e Pressure of water-vapour, *var* (0, 1), voltage (2*), charge on an electron (3), base of logarithms (1*, 2*, *passim*, see ε)

∂e Distance eastwards (1*)

f Relative humidity (0), pressure of wind, *var* (0), internal friction (2), pressure of saturated vapour (1), frequency (2*), various functions (1*)

g Acceleration of gravity (1*, 2*, *passim*)

g Preferably g, gramme (1, 2)

h Height or thickness, *var* (0, 1), quantity of heat, *var* (0, 2), height above mean sea level (1*); total heat of liquid (2*)

h Preferably h, Planck's constant (3)

i $\sqrt{-1}$ (*passim*), angle of incidence (0, 2), subscript for arbitrary height (1*), electric current (2*)

j Angle of refraction (0), special co-ordinate in soil (1*)

k A constant or coefficient (0, 2), an angle in optics (0), thermal conductivity (1*), thermal diffusivity (2*, see *K*); kilo (2*).

l Length, distance (1*); self-inductance (2*).

l Preferably l, symbol of length in dimensional equation

l, *m*, *n* Direction cosines (4, 5, 6).

m Mass, *var* (0); mass of an electron (3); integral number (0, 6), momentum per volume (1*); index (2*); mass of water-vapour in unit mass of moist air (1), magnetic pole (2*), mutual inductance (2*); milli- (2*).

m Preferably m, symbol of mass in dimensional equation

$m\mu$ Millimicro- (2*).

n Integral number (1*, *passim*); index (2*); volume in gas-equation (2), frequency (2*), ratio of specific heats (2*).

∂n Distance northwards (1*).

o

p Pressure, *var* (0, 1, 1*, 2), in kg/m² (7); vapour-pressure of liquid or saturation pressure (2*); pico 10^{-12} (2*).

q Vapour-pressure (0); horizontal temperature-gradient (0); electric charge (2*), "dryness fraction" or "quality" of mixture of liquid and vapour (2*)

q Preferably q, constant in pilot-balloon formula (0, 1)

r Radius-vector (1*, *passim*); elevation due to refraction (1), angle of refraction (2), correlation coefficient (0, 1, 1*, 6); relative humidity (1), resistance (2*).

s Density of water-vapour (0); horizontal pressure-gradient (0); diffusivity of soil for temperature (1*); specific heat of vapour at constant volume and specific heat of liquid or solid (2*); salinity of sea-water (1).

t	Time, *var* (1*, *passim*); temperature, *var* in various units (0, 1, 2); temperature in °C (2*).
T	Preferably T, symbol of time in dimensional equation.
tt	Temperature tercentesimal, *var* (0).
u	Velocity (2*), velocity-component in Cartesian and cylindrical co-ordinates, *var* (5); thermal capacity per volume (1*).
v	Velocity-component in Cartesian co-ordinates, *var* (5); velocity (0, 1*); specific volume, *var* (0, 1, 2), vapour-pressure in mb (1), specific volume of liquid (2*); voltage (2*).
w	Velocity-component in Cartesian co-ordinates, *var* (5); density of water (0); mass of water-substance per volume (1*), energy and work (2*).
x	Horizontal co-ordinate, *var* (1*, *passim*), deviation of X from mean (0); water-vapour associated with unit mass of dry air (0, 1), reactance (2*).
y	Horizontal co-ordinate, *var* (1*, *passim*), deviation of Y from mean (0).
z	Vertical co-ordinate, *var* (*passim*), impedance (2*); depth in ground (1*).

GREEK UNCIAL

Γ	Geopotential (0, 1); radiant energy absorbed at interface per area and per time (1*).
Δ	Small increment (1*), standard density of dry air (0).
Θ	Latitude (1); tercentesimal temperature (0), eddy-heat per mass (1*).
Λ	
Ξ	Mass of water evaporating from interface per horizontal area and per time (1*).
Π	
Σ	Sign of summation (1*, *passim*).
Υ, Φ	Relate to vertical velocity in the stratosphere (1*).
Φ	Entropy of vapour (2*); absorption band (2); magnetic flux (2*).
X	Absorption band (2).
Ψ	Absorption band (2); pressure in water in soil (1*).
Ω	Absorption band (2); ohm (2*), $-2\omega \sin\phi$ (1*); angular velocity (0).

GREEK MINUSCULE

α	Angle (*passim*); right ascension (4); phase of maximum (0, 6); angle of deflexion of surface-wind from gradient (0, 1*); coefficient relating to entropy (1*); specific volume (1); coefficient of absorption (1).
β	Lapse-rate of temperature (0); gradient of superposed field (0); angle due to frictional force (1*); coefficient relating to entropy (1*); coefficient of absorption (1); latitude (4).
γ	Ratio of specific heats (0, 2, 2*); gradient (1*); temperature-gradient (1); electric conductivity (2*).
γ_p	Pressure-gradient (1*); γ_{v1} velocity of gradient-wind (1*); γ_p, γ_v thermal capacities per mass (1*).
δ	Declination (4); finite difference operator (1*, *passim*); ratio of two specific weights (7); a coefficient of absorption (1); logarithmic decrement (2*).
∂	Symbol of partial differentiation (*passim*).
ϵ	Modulus of decay (0); a small correction (1); specific gravity of water-vapour (1); energy per mass (1*); change of translational molecular energy per 3·66 T_k (1*); factor depending on entropy (0).
ζ	Vorticity (0); zenith distance (1*).
η	Coefficient of viscosity (2); emissivity (1*); absorptance of stratum (1*); efficiency (2*).

θ Polar co-ordinate, *var* (*passim*), co-latitude (5), zenith distance (4), temperature, *var* (1, 2); temperature absolute (1*), potential temperature, *var* (0, 1), coefficient of conduction of heat (1*); temperature reckoned from absolute zero or from freezing point (2*)

θ_v Virtual temperature, *var* (1).

ι Eddy-conductivity in light winds (1*).

κ Coefficient of conductivity, eddy-diffusion, etc (0, 1, 2), electric inductivity (2*) molecular diffusivity (1*), ratio of specific heats at constant pressure and constant volume (1*).

λ Wave-length (0, 2), latitude (1), longitude (0, 1*), longitude always eastwards (1*)

μ Micron (1*, *passim*), index of refraction (0, 2), coefficient of viscosity (2, 7), joint mass of vapour, water and ice per mass of atmosphere (1*), permeability (2*), micro- (2*)

$\mu\mu$ Millimicron (1*, *passim*), pico- 10^{-12} (2*)

ν Coefficient of viscosity (1), kinematic viscosity (7), frequency (2), mass of liquid water per mass of atmosphere (1*).

ξ Turbulivity (1*).

ξ, η, ζ Departures in co-ordinates of position (5)

o Eddy-viscosity (1*)

π Ratio of circumference of circle to diameter (*passim*)

ρ Density of air (0, 1, 1*, 2, 7), absorption band (2), resistivity (2*), density (1*, 2*)

σ Stefan's constant (0, 1, 1*, 3), surface-charge (0, 2, 3), absorption band (2), standard deviation (0, 6); specific heat of air (0); entropy per mass of atmosphere (1*)

τ Absorption band (2); period (0, 6); potential temperature (1*)

υ Internal energy per mass of atmosphere (1*).

ϕ Polar co-ordinate, *var* (*passim*), latitude (0, 1, 1*, 4); zenith distance (1), phase angle (0, 6); entropy, *var* (0, 2); entropy of liquid (2*); gravity-potential (1), velocity potential (1)

χ In theory of stirring (1*).

ψ Stream function (5); ratio of two pressures (7); gravity-potential (increasing upwards) (1*); longitude (1).

ω Absorption band (2); angular velocity of the earth's rotation (0, 1, 1*)

$\omega_1 = \frac{d\omega}{dt}$ Angular acceleration, *var* (1*).

LIST OF WORDS

used in special senses or not yet incorporated in the *New English Dictionary* that have been found convenient for the avoidance of misunderstanding or for the sake of brevity

For winds (Introduction to the *Barometer Manual for the Use of Seamen*)

Geostrophic wind: that part of the horizontal component computed from the barometric gradient which is dependent on the rotation of the earth.

Cyclostrophic wind: that part of the horizontal component computed from the barometric gradient which depends on the radius of the small circle representing the direction of motion of the air at the moment.

Anabatic wind (Greek for wind going upward): the motion of air on a slope exposed to the warming influence of the sun.

Katabatic wind (Greek for wind going downward): the downward motion of air independent of the barometric gradient on a slope which is cooled by terrestrial radiation or by snow or ice.

For rate of variation of meteorological elements with height

Lapse-rate (*Meteorological Glossary*): rate of loss with height—generally of temperature: in place of gradient which from its origin should mean the fall of temperature along a horizontal line, a quantity of some importance but not much used in practice.

Counterlapse (Vol. III): the reverse of lapse, the recovery of temperature with height, a substitute for the word inversion which is suggestive of something "the wrong way up" unless the words "of vertical temperature gradient" are included.

For the specification of the atmosphere

Millibar (V. Bjerknes): approximately one thousandth part of a "normal atmosphere," a multiple of the c,g,s unit in which all measures of pressure should be expressed in the course of unavoidable "correction," whether the instrument be graduated ostensibly in inches or millimetres, as part of the comity of the physical sciences.

Geodynamic metre (Upper Air Commission): a practical unit for the expression of geopotential representing the "lift-effort" or energy required to lift unit mass from one point to another in the earth's gravitational field.

For the main divisions of the atmosphere

Stratosphere (Teisserenc de Bort): the region of the atmosphere, beyond the troposphere, in which there is little change of temperature with height.

Troposphere (Teisserenc de Bort), the region of the atmosphere from the ground upwards within which there is notable change of temperature with height, sometimes positive sometimes negative, tending towards the limit of adiabatic change for gaseous air at the tropopause

Tropopause (E. L. Hawke, *Meteorological Glossary*) the region which marks the upper limit of the troposphere and the lower limit of the stratosphere at which the lapse-rate of temperature shows a notable transition from a large positive value to one which is generally insignificant and sometimes reversed

In place of certain uses of the word "temperature" (Vol. III)

Thermancy: to indicate the property of a body upon which the energy of its radiation depends, and which, in the case of a gas, is a numerical expression of the translational kinetic energy of the molecules contained in unit mass. Absolute temperature is the customary expression, but the word "temperature" is claimed by those who "understand it" only when it is expressed in degrees Fahrenheit or Centigrade.

The thermancy of a gas at the temperature of n °C can be expressed with close approximation as $273 + n$, which brings it into easy relation with the tables of physical constants; otherwise the thermancy of air at the freezing point of water might be set at 1000, and an universal measure of temperature deduced from it, as suggested by A. McAdie. In Vol. III the thermancy is expressed provisionally as "temperature on the tercentesimal scale."

For the study of the thermodynamics of the atmosphere.

Potential temperature (von Bezold): the figure obtained when the observed temperature is "reduced" by adiabatic process to a standard pressure

Potential pressure (Vol. III): the figure obtained when the observed pressure is "reduced" by adiabatic process to a standard temperature

Megatemperature (Upper Air Commission): the potential temperature obtained by "reducing" the observed temperature to "standard pressure" of 1000 mb.

Tephigram (Vol. I): the curve, with temperature and entropy as co-ordinates, which represents the condition of the environment traversed by a sounding-balloon, an aeroplane, kite or other means of recording pressure and temperature.

Depegram (Vol. II): the curve on the same diagram, the temperature at any point of which represents the dew-point corresponding with the same pressure on the tephigram.

Liability (Vol. III) of the environment, indicated at a point of the tephigram, expresses the amount of energy that might be developed by the action of the environment on unit mass of air at the point with the temperature and pressure of the environment and with a specified condition as to humidity

Underworld (Vol. III): a portion of the earth's surface separated from the rest by intersection with an isentropic surface *rising* from its boundary.

THE PHYSICAL PROCESSES OF WEATHER

There is a curious similarity between meteorology and medicine which was expressed perhaps in past times by the astrological ideas of the relation of the macrocosm, the order of the universe, to the microcosm, the order of the human body. In more recent times the analogy finds recognition in various ways. V. Bjerknes, a natural philosopher, writes of the study of the meteorological situation as "diagnosis," and the precalculation of future states (of the atmosphere) as "prognosis." Mr Rudyard Kipling made some play with the astrological method of treating diseases in a copyright speech at a dinner of the Royal Society of Medicine. With the affectation of omniscience which sits so charmingly on the shoulders of a successful writer of fiction Mr Arnold Bennett[1] accentuates the analogy between meteorology and medicine in a periodical which has much larger circulation than that of meteorological discoveries. Both medicine and meteorology are of personal interest to everybody; the natural consequence of this universal interest is a gradation of the contributions which are offered for the presentation of either subject, in almost insensible steps between treatment on the most rigorous scientific lines and compositions which amount to sheer quackery, whether conscious or unconscious. These common characteristics of the two sciences are incidental to a similarity which is of greater scientific interest. The meteorologist, as we have already pointed out, must take his facts as he finds them in the life-history of weather, and endeavour by co-ordination and analysis to bring them into relation with the laws of nature which physicists and chemists have elaborated; and, in like manner, the student of medicine must take the facts and functions of the human body as he finds them in the life-history of man, and bring them into relation with the same laws of nature. In both sciences the facts are subject to the control of physical laws; in either, cases of similarity may occur; but in neither can any occurrence be repeated, no matter how frequently similarity may be observed.

In that lies the essential difference between the observational and the experimental sciences. That part of the science of medicine which concerns itself with the physics, chemistry and dynamics of the human body is called physiology, and thereby is introduced a subtle distinction between the physics of a living organism and the experimental physics of a laboratory.

Aerology is the special name, if any, for the part of the science of meteorology that deals with the control exercised over weather by the laws of physics, chemistry and dynamics; and it is well to keep in mind the essential difference between the sciences which are concerned entirely with experiment under

[1] "Men of science know no more about so-called inorganic matter and the mysterious antics thereof than doctors know about the human body. The merit of the best of them, like the merit of the best doctors, is that they know they don't know. A few know they never will know; which is an even greater merit."

the personal control of the operator, and those in which experiment can be used only to illustrate and account for observation.

We might indeed have profited by the analogy to which we have drawn attention by giving to this volume the title "The Physiology of Weather" as defining the attitude which meteorologists have to adopt towards experimental physics. We have felt however that to do so might convey the impression that we were proposing to regard weather as the expression of a living organism. Although the weather has many characteristics that are suggestive of vitality we have thought it best to avoid that impression.

As far as may be, we desire to give an insight into the physical processes that are operative in the control of weather. Our purpose is in fact to call the attention of the reader to the processes which can be recognised as physical, in the hope that he will be sufficiently interested to seek for any additional guidance that he may find necessary in the recognised treatises on the different parts of the subject. The achievement of that purpose implies the selection of a number of subjects from the recognised text-books of physics. Our presentation may be incomplete and disjointed, and for that reason a suggestion was made to define the scope of the volume with the title "Miscellanea physica," but that was found to be more recondite than wise.

WAVE-MOTION

The exposition of the subjection of the phenomena of weather to the control of physical laws begins conveniently with the consideration of wave-motion. Starting from the tidal wave which gets round the earth in about twenty-five hours, a maximum speed at the equator of 38,000 kilometres per day, and the visible waves of water which may travel with a speed of some 1500 kilometres per day and are indeed a natural demonstration of the mechanical energy of weather, we pass on to the suggestions of wave-motion of the same character in air, to the travel of sound and then to the reception and disposal of the vast amounts of energy in waves received from the sun which form the basis of all the various aspects of the Science of Meteorology.

Among the common features of the atmosphere which can be cited as illustrations of the laws and principles of physics few, if any, are more striking or more likely to excite curiosity than those which are concerned with light and sound. The blue sky, the red sunset and sunrise with their transient green ray, the lowering cloud with its silver lining, the sun drawing water, the fleecy cloud with its patch of iridescent colour, the mysterious halo, the mock sun, the distortion of the enormous orbs of the sun and moon at rising and setting, the crepuscular bands across the sky, and before all the rainbow with its message of hope for fine weather, the flash of lightning, the mysterious roll of thunder, the roar of the rushing wind and the fickleness of distant sounds are every man's experience and the subjects of every man's inquiry.

All these are regarded, by those who know, as belonging in some form or other to wave-motion. In the introductory portion of volume II we have

displayed a diagram of the fundamental properties of waves, their wave-length or their frequency and the rate of travel. In that diagram very small space indeed was allotted to the waves which are held to account for the behaviour of light and a space fifty times as large to other waves electric waves, which must be regarded as indistinguishable from light-waves except for the fact that the mechanism of our eyes is not adjusted to use them for seeing. All the waves enumerated in that diagram are regarded as being waves in the "aether," a medium which has been invented to account for the transmission of waves, to provide, as the late Lord Salisbury said, a nominative case for the verb "to undulate." Whether the aether has in fact a real existence or is a figment of scientific imagination it is certain that the behaviour of light, which is illustrated by the atmospheric phenomena that we have mentioned, has been explained more clearly than by any other method as the behaviour of waves travelling with an absolute velocity of nearly 200,000 miles a second, or 26,000,000,000 kilometres a day through an imponderable aether pervading space. We shall ask the reader to regard all the phenomena represented by that diagram as depending upon wave-motion.

> If we confine our attention to optical phenomena, the waves are all-important and we do not have to consider anything else, hence we can be content with an undulatory theory: if we study the electrical properties we are concerned with the energy and may be content with a corpuscular one, where attention is concentrated on the carriers of the energy. (Sir J. J. Thomson, *Beyond the Electron*, C.U. Press, 1928, p. 25.)

Sound, too, is definitely proved to travel as wave-motion, not in the imponderable aether but in the real atmosphere. It is not quite the same kind of wave-motion as that which is invoked to explain the behaviour of light. The record of sound received and preserved in the gramophone is a mechanical effect whereas the record of light which is preserved on a photographic plate is a chemical effect. The actual properties of the motion which constitutes sound have been subject to more thorough investigation than the supposed motion of the aether; and in consequence physicists are able to say with confidence that the travel of sound through the atmosphere is accounted for by the "elasticity of the air," which provides facilities for oscillation of the particles affected, backwards and forwards alternately, in the line of travel. While it is travelling, the sound consists of a succession of phases of alternate compression and rarefaction of the air, producing waves which have been made visible, at least photographically, by special contrivance. They travel at a speed which is proportional to the square root of the ratio of the elasticity of the air under very sudden compression to its density. The elasticity in those circumstances is represented by the familiar meteorological quantity pressure multiplied by a factor γ which is the ratio of the two specific heats of air (at constant pressure and constant volume respectively) and is equal to 1·40.

The velocity of travel of sound is in consequence proportional to the square root of the temperature of the air traversed. At the ordinary temperature of the air 290tt, it has the value of 342 m/sec, 1122 feet per second, 30,000 kilometres per day.

Wave-motion offers the most mysterious examples of the transmission of energy from one region of the earth or of the universe to another. By some process at present imperfectly understood a "train of waves" is set up in the transmitting medium, seismological waves in the earth, tidal waves or gravity waves or compression waves in water or air, capillary waves in water, electric waves including light waves and other waves of similar character in the hypothetical medium of transmission called the aether as set out in the diagram of waves, fig. ii of vol. II.

By a train of waves we understand a succession of similar waves following each other in rhythmical order with definite velocity along the surface of the earth or through its thickness, along the surface of water, along a surface of discontinuity in the atmosphere, through the atmosphere or through space. The "shape" of the wave travels along its medium with an appropriate velocity, while any particle that has taken part and is taking part in the transmission describes an "orbit" which it repeats as the successive waves of the train affect it. Where the medium is uniform in all directions the transmission is in straight lines, where the medium is varied a bend takes place, continuous variation in the medium means curvature in the line of transmission. A surface of discontinuity involves transmission along the surface, perhaps all round the earth. No energy is spent in the transmitting medium when the train is once established except that which is represented by the effect of internal friction (viscosity) of the medium, if any. The energy of the wave is passed on from particle to particle in a manner which is quite easily described but by no means easily explained.

The orbit of a particle which is taking part in the transmission may be simple or complicated. The simplest orbit is that of waves of sound through air of uniform composition and temperature. In that case the particle affected oscillates backwards and forwards along the straight line of transmission: the waves are then called longitudinal. Longitudinal waves occur in other "elastic" media, such as earth or water; a separate law of transmission applies to each kind of elasticity, longitudinal or transverse. Light waves also exhibit phenomena corresponding with simple linear oscillation but transverse to the line of transmission instead of being along the line. The motion of the particles in light is regarded as analysable into component oscillations at right angles to each other and to the direction of motion of the wave. The orbit may therefore be rectilinear, when the two components have the same phase, or it may be circular or elliptic, when the phases of the component oscillations are not identical.

Polarisation

Some substances like tourmaline or Nicol prisms have the remarkable property of "polarising" light, i.e. of being transparent in one position and quite opaque in another position for light which has passed already through one plate or prism. Allowing that in ordinary light the "particles" of the incident beam have an elliptical orbit, this astonishing property is explained

by supposing the first plate to be transparent to the component along one line and opaque to the component at right angles thereto, so that what falls upon the second plate is already in rectilinear vibration. Light which is transmitted by particles in rectilinear vibration is said to be "plane polarised" a name that sticks, though linear polarisation would probably express the idea better; but light with components at right angles may be elliptically polarised, circularly polarised or plane polarised, the motion being in every case confined to a plane at right angles to the line of transmission. Ordinary light may be regarded as consisting of a discontinuous succession of trains of waves representing successive quanta of energy, each train being separately unrestricted as to the orbits of its particles. Polarisation is exhibited only by light waves after they have been passed through some filtering medium that absorbs all the energy except that corresponding with the linear oscillation which the filtering medium can transmit.

The particles of gravity-waves in water or air may also have elliptical orbits but in that case one of the components is longitudinal, i.e. in the direction of motion, and the other vertical.

Rectilinear transmission of energy by waves and the law of inverse square

One of the most striking features of wave-motion is the transmission of the energy in straight lines. The shadows formed by opaque objects in a beam of sunlight are the most familiar example. In fact in accordance with modern views a straight line might be defined as the path of a beam of light, although there are cases in which the unsophisticated reader may have some difficulty in reconciling it with what he understands by the shortest distance between two points. From that principle, assuming that the medium of transmission is perfectly uniform, we may easily understand that the energy of any form of wave-motion originating in a point will spread out into a sphere with the point as centre and with a radius which increases with the velocity of transmission, just as though the energy belonged to a limited number of material particles projected in all directions from the point with the velocity appropriate to wave-motion in the medium. Thus the intensity of the energy per unit of area at any distance is like the force of gravity inversely proportional to the square of the distance from the point of origin.

That such a distribution is possible with energy that is expressed as wave-motion can be inferred by watching the spread of waves which originate from the point of disturbance of water caused by dropping a stone in an undisturbed surface. In order that the experiment may properly illustrate the principle there must be no obstacle in the path of any part of the advancing wave. An obstacle in the way spoils the regularity of the advance in its immediate neighbourhood and part of the energy is devoted to disturbing the medium behind the obstacle.

A proper undulatory theory takes account of such secondary disturbances;

they are included under the name of diffraction; the theory claims that the front of the wave can be regarded as made up of an infinite number of independent elements of disturbance each distributing its energy in independent wavelets, and is able to show that if the energy sent out by any element, at any angle θ from the normal to the wave-front, is related to the energy sent out along the normal by the factor $\cos\theta$, the combined effect of all the elements of the complete wave-front is the same as if the energy were all transmitted in straight lines with the inverse square law. The proposition of the rectilinear propagation of wave-motion is an essential part of the undulatory theory of light. The setting out of the proposition will be found in any account of the undulatory theory; reference may be made to Glazebrook's *Physical Optics*[1]. The proposition is not limited to light but applies to wave-motion in general. Any form of wave-motion will furnish illustrations of the rectilinear propagation of energy and the diffraction caused by obstacles.

The reconciliation of the undulatory theory and diffraction with the transmission of light in straight lines and orderly reflexion and refraction is a step of far-reaching importance towards the apprehension of the physical nature of the universe. It justifies us in associating together the breakers on Land's End, the crimson glow of a cloud in the evening sky and the invisible waves which, without any leave asked or given, pass through our homes and our bodies and by licence of the postmaster-general convey to us the prospects of to-morrow's weather—it enables us to treat all these things either as bundles of rays suggestive of corpuscular travel or as the effect of a train of wave-fronts with all the incidental consequences of diffraction, and we can, if we are so disposed, pursue the idea to the ramification of the tidal wave in an estuary a hundred miles from the sea. In the wave-motion last mentioned it is the energy conveyed by the travel of material that arrests our attention and reminds us that at both ends of the scale beyond the range of the ordinary sea-wave at one end and beyond the electron at the other the transference of energy is corpuscular, but in the open medium the motion is undulatory.

The most typical representation of wave-motion is the sine-curve in which ordinates at successive equal intervals from the starting-point correspond with the sines of angles with equal increment; the full angle of 360° corresponding with the length of the wave. We have already indicated in chap. XIII of vol. I the importance of the sine-curve in the analysis of the sequence of events, and there is no department of the science of meteorology for which the comprehension of a sine-curve is not required.

The sine-curve is the best illustration of the regular transmission of a *shape* as wave-motion: the shape transmitted in that case is the curve representing the fundamental component in harmonic analysis and is related to the horizontal or vertical displacement of a particle which describes a circle with uniform angular velocity. But the shape transmitted need not be and indeed seldom is a simple sine-curve. Harmonic analysis on Fourier's theorem enables us to resolve into a series of sine-curves of related periods, any shape whatever

[1] *Text-books of Science*, Longmans, Green and Co., London, 1883, chap. II.

that is repeated after a definite interval. The shape need not even be expressed by any finite number of harmonics. An almost infinite variety of shapes can be transmitted as wave-motion in a beam of sunlight in which the separate periods can be identified by suitable apparatus. When therefore we talk about a train of waves as represented by a sine-curve it should be understood that we are using the simplest form not because that is the most frequent or the most likely but because it presents the least difficulty in algebraical computation.

While we are thinking of changes which are represented by wave-motion and their laws we may take the opportunity of reminding the reader of the other type of change which is to be found all over the universe, namely, exponential change, the basis of the law of compound interest. For example, in the atmosphere when the temperature is uniform, pressure is proportional to $e^{-gz/Rtt}$, where z is the height, and to $e^{-E/R}$, where E is the entropy, and in similar conditions specific volume is proportional to $e^{E/R}$.

Between this logarithmic change with its perpetually increasing or diminishing value and cyclical change represented by the variations in the sine and cosine there is a curious association which is represented algebraically by the effect of the mysterious symbol $\sqrt{-1}$.

Thus as t increases e^t is a continuously increasing quantity and e^{-t} is a continuously decreasing quantity, $e^t + e^{-t}$ is the sum of two quantities one of which increases and the other decreases, but $e^{t\sqrt{-1}} + e^{-t\sqrt{-1}}$ is a periodic quantity, namely $2 \cos t$, and $(e^{t\sqrt{-1}} - e^{-t\sqrt{-1}})/\sqrt{-1}$ is also a periodic quantity, namely $2 \sin t$.

We can take the reader a step farther and combine the two expressions without much effort. $Ae^t(e^{t\sqrt{-1}} - e^{-t\sqrt{-1}})/2\sqrt{-1}$ will represent the "plane polarised" motion of a particle in the path of a train of waves when the amplitude of vibration Ae^t is gradually increasing beyond any possible limit, and $Ae^{-t}(e^{t\sqrt{-1}} - e^{-t\sqrt{-1}})/2\sqrt{-1}$ represents the same kind of motion which is gradually fading or decreasing in amplitude though it will take an infinity of time to reduce it actually to zero. So $A(e^t - e^{-t})(e^{t\sqrt{-1}} - e^{-t\sqrt{-1}})/2\sqrt{-1}$ represents two trains of waves in opposite phase passing the affected particle in the same direction, one increasing in amplitude without limit and the other fading. Increasing without limit is not a common occurrence but fading in periodic motion is common enough. The e^{-t} indicates what is called a coefficient of damping because the quantity affected by it is fading all the time. Curiously enough in all these calculations e is a number which cannot be expressed by a finite number of figures in the ordinary decimal notation, though it is indispensable for the construction of a table of logarithms. To the third place of decimals it is 2·718.

Wave-motion introduces us to the transference of energy and in that connexion we shall ask the reader's attention also to the logarithmic laws when we come to the relation of physical quantities to one another and their common relation to entropy which is of fundamental importance in the consideration of the energy of atmospheric changes.

THE PHYSICS OF THE ATMOSPHERE

Thus by the study of wave-motion in its simplicity or its complexity we are brought into quantitative relation with the general physical problem of the atmosphere, and the tracing of the transformations of energy in the sequence of the phenomena of weather. For that we require a working acquaintance with the application of the laws of thermodynamics to the various conditions of the atmosphere. We have taken the opportunity to put together the relations of the physical properties of the atmosphere to entropy and temperature in a form which enables us to set out the liability of the atmosphere at any time in respect of energy as disclosed by the results obtained from soundings by balloon.

We shall claim that the physical processes of weather are fairly well understood and from that point of view our knowledge of the physics of the atmosphere is generally sufficient for meteorological purposes. Having explored that province we shall take the opportunity of pointing out, as the conclusion of this volume, the bearing of some of our knowledge on the still unsolved problem of the general circulation. We are obliged to confess that our knowledge of the dynamics of the atmosphere is imperfect, singularly unaesthetic in its form and inadequate in its scope. The subject is really waiting for a novum organum.

There will remain for us therefore the examination of the methods for expressing the dynamical processes that are operative under the physical laws which this volume brings to account.

Some of the results of dynamical reasoning to which we have to call attention are already included in the volume which was published ten years ago as Part IV. Some prefatory chapters on the dynamical methods and some additional matter on the results of current dynamical theory in a new issue of that part as vol. IV will complete our representation of the subject.

*** The compilation of a volume so miscellaneous as the present one necessarily levies contributions from many authors. A large amount of information is naturally derived from the past issues of the recognised meteorological journals, the *Meteorologische Zeitschrift*, the *Journal of the Royal Meteorological Society*, the *Monthly Weather Review* and the *Meteorological Magazine*; and to these we must add the *Smithsonian Physical Tables* and the *Annals of the Astrophysical Observatory* of that Institution, the *Meteorological Glossary* and the *Dictionary of Applied Physics*. The principle upon which the structure rests is that it is best, so far as possible, to give an original author's own words and references to the source from which they have been derived. It is hoped that the references will not only be accepted as an acknowledgment by the author of his obligations for contributions to our common stock of knowledge, but also serve as an invitation to the reader to satisfy the natural desire for further information.

In addition to these obligations the author gladly acknowledges the assistance which he has received from friends who have read the work in proof; Sir Richard Glazebrook and Mr Sidney Skinner, colleagues for many years in the teaching of practical physics at Cambridge; Mr R. G. K. Lempfert, the first of a series of personal assistants at the Meteorological Office; Mr D. Brunt and Commander L. G. Garbett, R.N., associates in a later effort at the Imperial College of Science and Technology to represent for a class of students the application of physical laws and principles in the atmosphere.

In the suggestions which have arisen in the discussion of the text with one or other of these friends there has emerged a feeling of uncertainty as to the class of readers to whom this volume is offered. It is not a text-book of physics, nor yet a text-book of meteorology which assumes all physics and its auxiliary mathematics as the common possession of author and readers. Some things which cannot be regarded as easy are assumed and some that are not difficult are expounded.

Acknowledging the impeachment the author would plead that his purpose in writing is not that of the text-book writer, which may be succinctly described as saving his readers as far as possible the trouble of thinking, by going through that process for them; but to suggest that, comprised within the almost unpronounceable name of meteorology, there are a large number of subjects that readers will find quite interesting and worth their while to think about, and to indicate to them at least where and how food for thought can be found. Within the last half-century the pursuit of meteorology as a science has been to some extent accepted as a responsibility of government, and the amateur has to the same extent been exonerated from supplying the material for the study of the atmosphere of the globe. As may be gathered from vol. II of this Manual, students of the subject have become aware of the gradual and orderly compilation of a vast multitude of data, but the opportunities of thinking about them have not been extended equally with the material to be thought about, and part at least of the responsibility of converting scientific data into science still remains for the amateur or the leisured hours of the official.

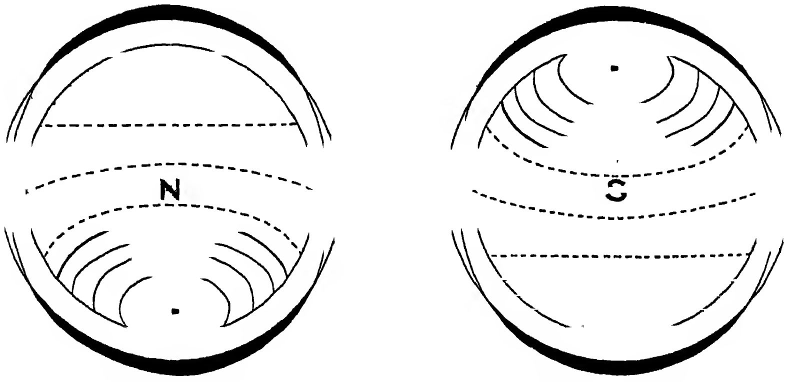
Nature as represented in weather is a little intolerant of organisation and classification and some parts of the subject, for example the stereography or the kinematography of clouds, belong to no official routine and offer an invitation to the enthusiastic amateur. There are many others which will suggest themselves to those who think about what has already been achieved, and this volume is in fact addressed to those who agree that "the books that help you most are those that make you think most."

CHAPTER I

GRAVITY-WAVES IN WATER AND AIR

The tides at this place [Funchal] flow at the full and change of the moon, north and south; the spring tides rise seven feet perpendicular, and the neap-tides four. (*The Voyages of Captain James Cook*, London, 1842.)

LUNAR TIDE ON THE EQUILIBRIUM THEORY



Northern hemisphere Southern hemisphere

Fig. 1. Adapted from fig. 29 of Sir G. H. Darwin's *Tides*, London, John Murray, 1898. The distribution of the displacement of the surface of a shell of deep water covering both hemispheres, computed for a tidal wave according to the equilibrium theory, adapted to the mode of circumpolar representation employed in this Manual for the distribution of temperature, pressure, etc.

Continuous curves indicate elevation of the water, interrupted curves depression.

The effect represented is the heaping up of water round a centre, marked by a dot in lat. 15° N immediately over which the influencing body (the moon) is supposed to be.

Counting as 2 the vertical elevation of the water at that point, concentric "small circles" round the central point show the positions of elevations measuring successively $1\frac{1}{2}$, 1, $\frac{1}{2}$, 0, four curves of continuous line. The last which marks the circle of no displacement is a thicker line. Beyond that, represented by interrupted lines, are successively the circles of depression of $\frac{1}{2}$ and 1 (the maximum depression) and again of $\frac{1}{2}$, a step towards another thick line for the second circle of no displacement. The incomplete curves of the one hemisphere are completed in the opposite hemisphere, the circles of elevation being centred at a point 15° S where the elevation is also 2, the antipodes of the first.

The profile of the wave (greatly exaggerated) along latitude 15° N is shown by the ovals surrounding the hemispheres—the elevated part is blackened. In order to realise the full effect of the displacement the oval curve with its protuberances must be thought of as rotated about its long axis.

For reasons which are explained in Sir G. Darwin's work the distribution of tidal water in this hypothetical wave does not agree with that of the observed tides in the open oceans.

The reader will find some interesting information on the subject of the equilibrium tide and its transformation into tidal waves on shore and in estuaries in a letter by A. Mallock, *Nature*, vol. CXXIII, 1929, p. 640, and in the works of G. I. Taylor or H. Jeffreys on tidal friction in shallow seas.

THE first form of wave-motion which we shall consider is that of water-waves. Water-waves are interesting because even the unaided eye can see something of what is going on in the process. The water-wave is indeed the starting-point of all scientific ideas of wave-motion, and by way of fixing those ideas in the memory we will remind our readers of some of the salient features of the water-wave. It is the more natural to do so because in many cases wave-motion in water is due to wind blowing along the surface, and wind blowing along water can be regarded as an example of a current of light fluid, air, passing over a heavy fluid, water; the difference of density of the two fluids is so great that except for some minor effects each fluid keeps its relative position; but there is no essential dynamical difference between the effect of air moving over water and that of an upper layer of light warm air moving over a lower layer of heavy cold air. In that case also waves like water-waves will be set up in each layer. They are quite different from sound-waves. Modern meteorology has to take account of such waves. It is partly for that reason that water-waves are brought here within our cognisance, but also partly because we can learn from the behaviour of water-waves in the neighbourhood of obstacles something about waves of light, and about such meteorological features as the transparency of the air, the translucency of mist and the opacity of other forms of matter.

The waves between air and water or between two layers of air are called "gravity-waves" because the force which controls their behaviour is the force of gravity upon the heap of water or air in the protuberant part of the wave.

The most conspicuous example of the gravity-wave is the tidal wave (fig. 1) which is caused by the differential effect of the attraction of the sun and moon, in successive phases of the moon, upon the rotating earth, and upon the water which lies upon it. In the open ocean it is expressed by a deformation of the surface of the water with a protuberance under the moon and its antipodes. In the more enclosed sea-basins the periodical heaping becomes the ebb and flow of a tidal wave, known on all shores, which, with an allowance for lag, keeps time with the moon and goes through its period in approximately 12½ hours.

Anyone who wishes to realise the power of human ingenuity and perseverance to unravel the mysteries of nature would be well advised to make some daily observations upon the ebb and flow of the tide and the variation in its range when he has the opportunity of spending a month or two at any point on the British coasts. He can then trace for himself the process of associating the familiar habit of the tide with the period of the moon's phase on the meridian, approximately 25 hours. He can trace also the fortnightly change from spring tides with their extremes of high and low water, to neap tides with lowest high water and highest low water, and back again, and its association with the sun's period of 24 hours. He may find therein a fascinating example of the amplitude of oscillation due to the combination of two oscillations of nearly equal period which, in the study of wave-motion, results in what are known as beats. He may find further interest in tracing the behaviour

of the tides at different points of the coast and trace the influence of the tidal streams which derive their energy from the tidal waves of the ocean.

The ordinary sea-waves that are caused by the wind have periods which are measured in seconds not in hours. Scientific people draw a distinction between waves of that kind and the ripples which belong to the ruffling of the surface of water by the wind. We shall not pursue the inquiry in that direction because, so far as we know, there is nothing between two layers of the atmosphere which corresponds with ripples on water. There are clouds which look like a rippled surface of water, and are sometimes called ripple-clouds, but from the point of view of experimental physics the appearance in the cloud is regarded as due to gravity-waves not ripples in the technical sense. The driving-force of true ripples is the peculiar feature of a bounding surface of water called surface-tension which causes a water-drop to be spherical, and also causes water to rise automatically in a capillary tube. We do not recognise any surface-tension between adjacent layers of air.

And yet we may use the occasion of the mention of ripples to remark that one of the peculiarities of wave-motion is the faculty which water has of carrying two or more kinds of waves simultaneously almost completely independent the one of the other. All kinds of trains of waves may be seen travelling along the sea-surface, each train keeping its identity unimpaired, a ground swell, a cross swell, wind-waves, reflected waves, and ripples, may all be seen making use of the same water. No doubt any particle of water can only be moving in one direction at one time, but the motion which represents the result of the several trains of waves is compounded to give a resultant motion by which each particle may be said to bear its part in each one of the component motions.

The combination of the movements of a particle under the simultaneous influence of a number of trains of waves is the basis of the principle of interference which is a fundamental principle of the undulatory theory of light. Interference in this sense can be seen in water-waves.

Interference (which one might be disposed to call non-interference) is characteristic of all forms of wave-motion, particularly of sound and light. The owner of a gramophone knows that the air can carry many sounds simultaneously and deliver each as though the rest were not there. In transmitting the sounds which constitute orchestral music the motion of the particles is far too complicated for description; but it exists, and the identity of each vibration transmitted thereby is unimpaired. It can be picked out to the exclusion of others by suitable apparatus. Similarly a beam of sunlight is a combination of an almost unlimited number of different waves each preserving its identity.

It is this preservation of identity in the complication of wave-motion which is the sustaining idea of the analysis of atmospheric changes into harmonic components to which reference has already been made; it is only a reader acquainted with the physical experience of the complication of waves who can be expected to have the patience necessary for pursuing a hypothetical period in a chronological series of figures.

E. H. u. W. Weber, *Wellenlehre*. Leipzig, 1825.
J. Scott Russell, *Report on Waves*, *British Association Report*, 1844.
Sir W. H. White, *A Manual of Naval Architecture*. 5th edition. London, John Murray, 1900.
Vaughan Cornish, *Waves of the sea and other water-waves*. T. Fisher Unwin, 1910.

The properties of water-waves which we wish the reader to have in mind are first the characteristic features of the motion of a particle which is affected by a train of waves, secondly the velocity of travel and thirdly the behaviour of waves which pass obstacles in their path.

We shall speak of trains of waves supposing each wave to be exactly like its predecessor and its successor. That, of course, can never be exactly true, the waves are either developing or decaying, and each individual wave is somewhat different from the others; but at sea, or even near the shore, the waves succeed one another with such apparent regularity that it is easy to think about a train of invariable waves. Watching a train of deep-sea waves we cannot fail to recognise the elevation of the water into ridges and furrows, the separation of either of which marks the wave-length; the height of the ridges above the furrows marks the height of the wave, and the rate of apparent motion the velocity of travel. As with all other waves the wave-length λ, the speed V, and the period τ of the cycle in seconds or its reciprocal the frequency n, i.e. the number of cycles in unit time, are related by the simple formulae

$$\lambda = \mathrm{V}\tau, \qquad \lambda \mathrm{n} = \mathrm{V}.$$

The things that are less easy to mark are the height and the shape of the surface as the wave travels, and the relationship between the two. They have been the subject of careful scrutiny for more than a century.

Hitherto when we have made any reference to wave-motion we have assumed that a travelling wave could be regarded as the result of "simple harmonic motion" and its profile represented by a sine-curve with the equation

$$z = B \cos \frac{2\pi}{\lambda}(x - \mathrm{V}t)$$

or a combination of harmonics of that kind. Ordinary water-waves cannot be so regarded. The ridges are steeper than the bend of a sine-curve and the furrows are more nearly flat. The undisturbed level is less than half of the way up from the bottom of the furrow to the top of the ridge. Moreover the particles, when they are transmitting the wave, do not move backwards and forwards in straight lines but describe closed curves,—circles or ovals. A satisfactory representation of the profiles of water-waves has been found in the shapes of the curves traced out by a point on the spoke of a rolling wheel and called by the family name of trochoid. Their shapes vary between that traced out by a point on the circumference of the rolling circle, which is known as the cycloid, and the straight line traced by the centre of the rolling circle.

Subject to some uncertainty due to the rotational character of the motion the accepted view of the nature of sea-waves is that due to F. J. von Gerstner[1],

[1] Horace Lamb, *Hydrodynamics*, Cambridge University Press, 4th edition, 1916, p. 412.

professor of mathematics at Prague 1789–1823, who wrote in the first years of the nineteenth century, and Rankine who belonged to the middle of the same century. It supposes that each of the particles which constitute a deep-sea wave describes in its turn a circular or elliptic orbit, and describes the same orbit time after time as successive waves pass over; the shape of the surface

THE TRAVEL OF A WATER-WAVE

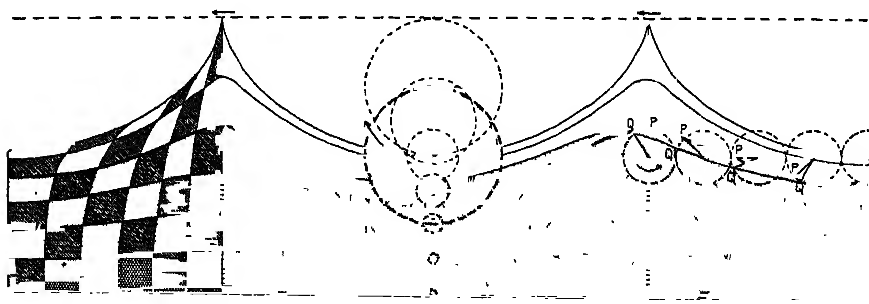


Fig. 2. A representation of the progress of a water-wave on the trochoidal theory of von Gerstner. (From the *Report of the British Association*, 1844, London, 1845.) The trochoid for the surface is shown by the thick wavy line. The travel of the wave is from right to left. The rolling circle (the same in diameter λ/π for each layer) is shown by the thick circle with an interrupted circumference, the level of undisturbed water by its centre. The travel of the wave is controlled by the roll of the circle from right to left, the orbit of a point in the surface is shown by the smaller concentric circle which is repeated in the panel on the right.

Each particle takes the same time to complete one revolution in its orbit. The elevation and depression of a particle of the water during the travel of the wave are controlled by the radius of the particle's orbit.

One step in the process of travel of the wave in the surface is marked by the change from the initial positions PP... to the positions QQ... by equal steps in the rotation of the particles indicated.

The progression of the wave at any level below the surface is carried out by rotation of the particle in a smaller circle.

The chequered figure on the left shows the distortion of the water in a vertical section.

Above the thick line which is supposed to mark the surface are two other surfaces which might be developed by the persistent action of the wind. The upper line represents the maximum height which a wave of that particular length can reach: if urged beyond that limit the wave must break.

To construct the curve of deformation for the layer at any given undisturbed level, take a point at that level as centre, draw the rolling circle by drawing a circle with the fixed length of radius, $\lambda/2\pi$, proper for the wave-length and velocity. On the vertical radius mark a point distant from the centre to indicate the amplitude appropriate to the depth. Set the rolling circle through a revolution with the selected point on the radius as a tracing point.

is defined by the difference of position of the successive particles in their orbits for any position of the wave. The actual curve of displacement of each layer of water for a wave with circular orbit is called a trochoid and is that described by a point on the spoke of a wheel which is supposed to roll along a line $\frac{1}{2}\lambda/\pi$ above the surface of the water or of one of the layers beneath of which the motion is sought. The centre of the rolling circle is in the surface

of the undisturbed water or of the undisturbed layer beneath; its circumference is the length of the wave, and the position of the point on the spoke is at a radius equal to the amplitude of the wave, that is one-half of the difference of level between crest and trough.

The travel is simply a matter of appearance due to the fact that the orbits of successive particles are so timed that each reaches the extreme of its amplitude in its turn, and in consequence the shape at the surface does actually travel with a definite speed although the particles which form it simply describe circles.

The motion is not entirely confined to the surface. Clearly the surface could not be depressed to the shape of the wave unless provision were made for disposing of the water displaced. The provision indicated is that each of the particles in any vertical describes an orbit the radius of which diminishes with the depth and becomes insensible at great depths. Hence each separate layer has its own trochoidal surface.

The motion of a particle under the influence of a train of waves of unchanging form which travels along the line x is

$$\mathrm{x} = A \cos \frac{2\pi}{\lambda}(x - \mathrm{V}t),$$

$$z = B \sin \frac{2\pi}{\lambda}(x - \mathrm{V}t),$$

where x is the horizontal and z the vertical displacement of the particle, λ is the wave-length, t the time at which the displacements are $\mathrm{x}\ z$, A the amplitude of the horizontal oscillation, B that of the vertical oscillation. In the conventional trochoidal wave the two amplitudes A and B are equal. The cyclic frequency n per second is the reciprocal of τ, the number of seconds in a complete period. It is evident that if we substitute for x, $x + \lambda$, and for t, $t + \tau$, nothing is changed, if $\lambda = \mathrm{V}\tau$ or $\mathrm{n} = \mathrm{V}/\lambda$.

In the consideration of sea-waves the sea is deep, and the waves are called deep-sea waves, if the wave-length is "small compared with the depth." The water will certainly be deep if the wave-length is only a tenth or twentieth of the depth.

With deep-sea waves, although the relation $\lambda = \mathrm{V}/\mathrm{n}$ holds, the velocity of travel V is different for waves of different length, the longer waves travelling faster than the shorter; the relation between wave-length and velocity has been expressed by a formula $\mathrm{V}^2 = g\lambda/2\pi$ which is applicable when the length of the wave is negligible compared with the depth of the sea in which the wave travels. A table of velocity in relation to wave-length expressing the formula is given below. The meaning of group-velocity is given later.

Wave-length in deep sea	5	10	25	50	100	150	200	250	300	metres
Wave-period ...	1·8	2·5	4·0	5·7	8·0	9·8	11·3	12·7	13·9	sec
Velocity of individual waves	2·8	3·9	6·2	8·8	12·5	15·3	17·7	19·8	21·6	m/sec
Velocity of the disturbance or 'group'	1·4	2·0	3·1	4·4	6·2	7·6	8·8	9·9	10·8	m/sec

The energy of waves

We may form an estimate of the energy of a train of waves, assuming its profile to be a curve of cosines represented by the formula

$$z = B \cos \frac{2\pi}{\lambda} (x - Vt),$$

from which it follows that the total energy of "motion" for a complete wave-length is $\frac{1}{4} g\rho B^2 \lambda$; and as the energy of motion is equal to the potential energy of gravitation due to displacement from the position of equilibrium, the total energy of the wave is $\frac{1}{2} g\rho B^2 \lambda$. On that understanding a section, a metre in width, of a water-wave with a difference of three metres between crest and hollow (amplitude 1·5 m) and a length of 30 metres would have an energy of $3{\cdot}3 \times 10^{12}$ ergs, about 33 metre-tonnes. Thirty such waves, a kilometre in width, would have energy equivalent to 10^{17} ergs or 10^{10} joules, which we have assigned in the table on p. xl of volume II as the normal equivalent of a flash of lightning.

When a deep-sea wave approaches the land along a shelving shore the motion is retarded at the bottom and the comparatively harmless energy of the wave-motion is transformed into the destructive energy of a moving mass of water in the breaking wave, which on occasion plays great havoc with the structures of the coast-line.

In favourable circumstances the energy expressed by great water-waves is very formidable. "At Peterhead, as an example, blocks of wall weighing 41 tons each have been thrown out of position, though no less than 37 ft below low-tide mark, and in the same section of blockwork a portion weighing 3300 tons was once shifted two inches without being broken." (R. L. Hadfield, *Daily News and Westminster Gazette*, 12 April 1928.)

It must be remarked that with the exception of tidal waves which are regularly periodic, and seismic waves which are few, the energy of water-waves is due to the action of the wind. It is a striking example of the gradual accumulation of energy to produce a force far greater than any that was exerted by the energy in its original form.

It will be evident that the elaborate structure of a wave of water cannot be created instantaneously, it has to be developed by repeated intensification of an original disturbance.

The rollers of Ascension

The most celebrated examples of trains of waves coming from a great distance and developing a devastating energy upon an exposed roadstead are afforded by the rollers of Ascension and St Helena where on occasions they form colossal breakers.

From the account given by Dr Otto Krümmel[1] it appears that the worst manifestations occur within the northern winter, December to April, when the North Atlantic is most subject to difference of temperature of sea and air.

[1] *Handbuch der Ozeanographie*, Band II, 2 Aufl., Stuttgart, 1911, S. 115.

They come from the north-west and strike without hindrance the roadsteads on the north-west side of the islands. In the southern winter they come from the south-west and are less strong. With the finest weather and no wind they may break with such force that houses are shaken to their foundations, and in earlier times loss of ships was very frequent. In February 1846 a large number of slave-ships anchored near the land were destroyed. Ships anchored in water deeper than 10 fathoms were uninjured but powerless witnesses of the destruction. The relative monthly frequency of rollers and of north-westerly storms in the North Atlantic, each referred to 100 for the month of maximum, is as follows:

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Rollers ...	3	2	1	7	25	43	62	100	40	9	1	5
NW storms	2	5	15	23	45	90	100	68	65	35	8	2

Similar phenomena are observed at Fernando Noronha, Angola and at Tristan da Cunha and various points on the shores of Africa.

The rollers at Ascension are thus described by Mr W. H. B. Webster, Surgeon of H.M.S. *Chanticleer*, 1829:

"One of the most interesting phenomena at Ascension are the rollers; in other words, a heavy swell producing a high surf on the leeward shores of the island, occurring without any apparent cause. All is tranquil in the distance, the sea-breeze scarcely ripples the surface of the water, when a high swelling wave is suddenly observed rolling towards the island. At first it appears to move slowly forward, till at length it breaks on the outer reefs. The swell then increases, wave urges on wave, until it reaches the beach, where it bursts with tremendous fury."

(*Africa Pilot*, Part II, sixth edition, 1910, p. 269, Hydrographic Office, Admiralty.)

A somewhat similar description is given by A. R. Wallace of heavy breakers in the bay or roadstead of Ampanam in the Lombok strait, near Java, and Humboldt describes the sudden invasion of the generally peaceful coast of Peru by a dangerous swell with waves 3 or 4 metres high. Similar phenomena are observed on the coast of Sumatra, and at Paumotu, Low Archipelago, on the south-west side.

These are examples of the transference of the energy of wind over vast distances by waves. The waves which develop into rollers are deep-sea waves the properties of which have been the subject of successful theoretical investigation by G. G. Stokes[1] and others. They form a normal section of the text-books of hydrodynamics. We may conclude that the amplitude of the orbit of the particles of water, the motion of which forms the wave, diminishes with the depth according to the logarithmic law that equal fractions of the amplitude are lost for successively equal steps in depth; thus the amplitude at a depth of one-ninth of the wave-length is one-half of that at the surface, at two-ninths it is one-quarter and so on; thus an ocean-wave 600 feet (183 m) long and 40 feet (12 m) from hollow to crest will have a height of about 5 feet at 200 feet ($\frac{3}{9}$ of the wave-length) below the surface.

[1] Sir George Gabriel Stokes, *Memoir and Scientific correspondence, selected and arranged by Joseph Larmor*, 2 vols., Cambridge University Press, 1907.

Wave-velocity and group-velocity

The travel of long sea-waves is in the direction of the train and continues in the same direction far beyond the region from which the energy of the motion is derived. The velocity of travel according to the table on p. 15 depends upon the length of the wave, and a swell which produces breakers of quite notable character upon a distant shore may travel in advance of the meteorological conditions which gave rise to the original waves. Many years ago Stokes[1] called attention to this possibility and suggested its use in forecasting the arrival of advancing cyclonic systems. The existence of a swell on the shore is indeed commonly recognised as a precursor of change, but no special attention has been given to the study.

It is however germane to the question of the difference between the velocity of a wave in a train of uniform waves and the velocity of a "group" of waves which has been treated by Osborne Reynolds, and the third Lord Rayleigh, and which finds its application in all cases of wave-motion comprising waves of different velocity.

If it should be desired to affect a distant point by means of a train of waves maintained in operation at the station of origin we have to bear in mind that before communication is established at the receiving station the energy necessary to provide the connecting train of waves must be provided. It can only be derived from that of the earlier waves and these are in consequence sacrificed in setting up the motion.

The conclusion arrived at is that the establishment of full communication by the train of waves travels with what is called the group-velocity, which is one-half the velocity with which the individual wave travels when the full communication is established. If we confine our attention to waves which form an isolated group the waves must be regarded as travelling through the group and losing their energy on emergence in the formation of a new front to the group. In the table on p. 15 the group-velocity for waves of different period is entered in the last line.

The height of waves

The height of a wave, the difference of height between crest and trough, cannot be deduced from the general theory. For a wave of sine or cosine form it is expressed by double the value of the quantity B of the formula (p. 13). Clearly some form of spiral growth not in accord with the normal formula for the final result is necessary in order to make a particle at rest develop motion in a circular or elliptic orbit which keeps its distance from the original position, that is, the position of rest. We have to depend upon observation for information about the height of waves.

Much attention has been paid to the subject by Lieut. A. Paris, a French naval officer, and Dr Vaughan Cornish. The latter, who has studied waves in many parts of the world, assigns 70 feet as the maximum height for sea-

[1] *The Marine Observer's Handbook*, M.O. publication, No. 218 (2nd edition), 1918, p. 58, quoting a letter, dated 12 Sept. 1878, from Sir G. G. Stokes to Capt. H. Toynbee.

waves; that would be somewhere near the height of the buildings in one of the principal streets of London.

Both these observers endeavoured to establish by observation a relation between the height of waves and the velocity of the wind observed at the time that the height was obtained. Such an endeavour evidently labours under some difficulty because the great wave is the integrated result of a wind of "long fetch," the "fetch" being the distance from the position where the wind and the wave started. Even if the wind were perfectly steady over the whole fetch, so long as it was using its energy to build up the wave it would have to go faster than the wave travelled, and it would only be when the condition becomes steady, when all that remains for the wind to do is to prevent the wave losing height, that a definite relation could exist. So it is not surprising that the two authorities mentioned come to somewhat different conclusions.

Lieutenant Paris made out that, roughly speaking, the speed of the wave was four times the square root of the speed of the wind. Dr Cornish, observing waves of long fetch, thought the speed of the wind was practically the same as that of the wave. The summary of results quoted by Dr Cornish[1] (from observations by Scoresby 1848, Paris 1867, Abercromby 1885, David, S. Indian Ocean, 1907, and himself) is as follows:

Date	Height of wave Mode m	Height of wave Max. m	Length m	Period sec	Wind m/sec	Locality
iv. 12	—	—	—	6·8	9	Caribbean Sea
i. 07	4·6	>6·1	61	—	11	" "
iii. 48	7·9	—	171	—	16	North Atlantic
xii. 00	8·8	13·1	—	—	20	" "
iii. 48	9·1	12·2	—	—	20	" "
xii. 11	>9·4	—	—	13·5	21	Bay of Biscay
xii. 98 } ii. 99 }	—	14 to 16 (at sea)	—	19 to 22·5	29 to 34	Dorset Coast
viii. 07	12 to 14	15·2	206	—	20	S. Indian Ocean
x. 67	9·0	>11·5	235	—	21	" " "
vi. 85	7·3	7·9	135	9	14	South Pacific

Dr Cornish's summary for the velocity of wind and waves between 25 and 77 miles per hour (11 and 34 m/sec) is that the velocity of the wave in m/sec $= 1{\cdot}56 \times$ period in seconds $= \sqrt{1{\cdot}56 \times \text{length in metres}}$ according to the trochoidal theory, so that the length is $1{\cdot}56 \times$ square of period. From observation he deduces that the height of the wave in feet is $0{\cdot}7 \times$ velocity of the wave in statute miles per hour; from which in c, g, s units we obtain:

$$\text{height of wave in metres} = {\cdot}477 \times \text{velocity in m/sec},$$

hence $$\text{length} \div \text{height} = 1{\cdot}342 \times \text{velocity in m/sec}.$$

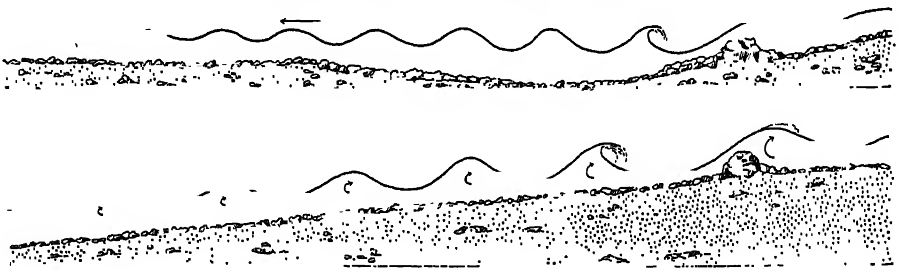
An explanation by Prof. Proudman of the development of a flooding wave on the south coast of England by sudden increase of pressure in an approaching line-squall on 20 July 1929, is given in a note by C. K. M. Douglas (*Meteorological Magazine*, vol. LXIV, p. 188).

[1] 'Ocean waves, sea-beaches and sandbanks,' *Journal of the Royal Society of Arts*, vol. LX, 1912, pp. 1105–10.

Standing waves in running water

In what precedes we have confined our attention to waves that travel through water otherwise undisturbed. In view of the possibilities of atmospheric waves of character similar to water-waves we ought not to omit a reference to the permanent waves which can be noticed in a flowing stream. The motion must necessarily be confused unless it can be adjusted so that the effect of a disturbance in the flowing stream becomes equivalent to a wave travelling up-stream with a velocity equal and opposite to the flow of the stream and therefore appearing stationary with regard to the spectator. Such stationary deformations of the surface of the running water are called standing waves.

STANDING WAVES IN FLOWING WATER



Figs. 3 and 4. (Scott Russell.) Profiles of standing or stationary waves in a stream flowing over an obstacle in a pebbly bed, (3) with variable slope, (4) with uniform slope. The first wave beyond the obstacle is a continuous "breaker." The note sounded by the falling water is "the tinkling of the brook."

The phenomena can be observed in any stream that flows over an irregular bed, and any notable obstacle in the stream causes a special succession of standing waves of diminishing height having their successive crests separated by a wave-length. It is upon the adjustment of the wave-length that the apparently stationary condition of the surface depends.

The forms of the waves in these figures are the same as those [of the travelling trochoidal wave in which the particles of the wave move in circular orbits while the profile travels with a certain accentuation of the shape in consequence of the limited depth of water] with this difference only, that the latter were moving along the standing water with a uniform velocity while those in figs. 3–6 are standing in the running water. The generating cause in this case is a large obstacle or large stone in the running stream. On this the water impinges; it is heaped up behind it; it acquires a circular motion which is alternately coincident with and opposed to the stream; the water having once acquired this circular oscillating motion in a vertical direction retains it, the water is alternately accumulated and accelerated, and thus standing waves are formed, as shown in figs. 3 and 4.

(*Report of the British Association for the Advancement of Science*, 1844, p. 389 and plate LV, London, 1845.)

The adjustment of the wave-length to the flow of the stream and the size of the obstacle is illustrated by these two figures.

Figs. 5 and 6 exhibit a remarkable case of the coexistence in one stream of two sets of waves moving with velocities differing in about the proportion of two to three. [Fig. 5 represents the plan of a flowing stream with a ledge at one end over which the whole stream flows.] On one side of a stream there projected a ledge of rock over which fell a thin sheet of water into a large pool, nearly still, without generating any sensible wave. On the opposite side a deep violent current was running round the obstacle with great rapidity. The middle part of the channel was occupied by a large boulder, over which also a stream flowed, generating standing waves with a smaller velocity. These waves [which are represented in fig. 6] are also remarkable for non-diffusion, as they will preserve their visible identity to a great distance without being dissipated. (*Ibid.* p. 389.)

STANDING WAVES IN FLOWING WATER

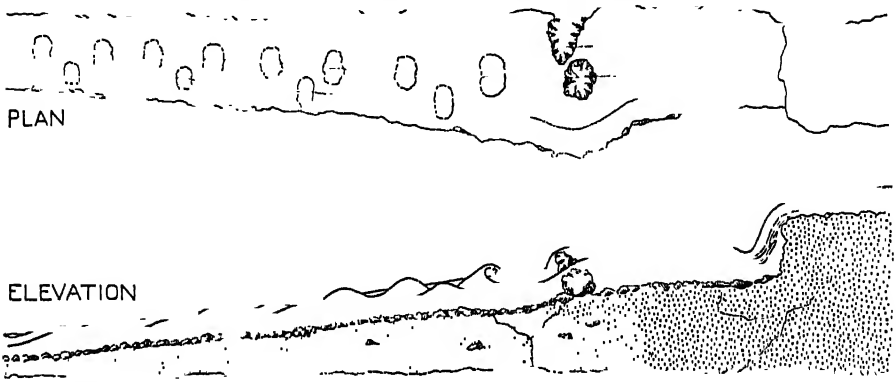


Fig. 5. Plan of a stream with cliff on the right, a projection from one side, and an obstacle in the fairway.
Fig. 6. The profile of the wave-motion along the thin lines set out in plan in fig. 5.

The process of wave-formation which is here described is exhibited on the grand scale in the rapids of Niagara below the falls. It must be followed, *mutatis mutandis*, wherever a current of air flows over an irregular surface.

The tinkling of the brook

It will be noticed in figs. 3, 4 and 6 that the first wave after the obstacle is a breaker. The vertical displacement of the water exceeds the limit which can be maintained by the adjustment of rotation to wave-length.

The broken water is, in consequence, always falling from the crest of the stationary wave into the flowing water beneath. A trail of bubbles in the stream marks the effect which is the same as if water were constantly allowed to drip from a tap into the stream. When the flowing stream is not too boisterous only a few obstacles are important enough to cause breaking waves, perhaps only a single one. In that case the tinkle of the water continuously dropping from the breaker has a musical ring about it which is far from unpleasant. Part of the energy which has proved too much for its accommodation in a water-wave finds expression in quite another kind of wave-motion. The standing wave becomes the mouthpiece of the babbling brook. A very

cursory examination of the brook will show which of the obstacles is taking part in this natural concert.

The same kind of motion in the atmosphere might substitute for the water that falls from the breaker a shower of rain, the duration of which might correspond with the duration of that form of wave-motion in the air.

Obstructions in the path of waves

We have just been considering the production of waves by obstacles in a stream which otherwise would flow smoothly and uniformly, and we have now to consider the effect of obstacles in the undisturbed water upon the waves which would otherwise preserve their course in a direct line.

We will begin by imagining a perfectly smooth vertical wall of unlimited depth in face of a train of regular waves. It is not likely that the reader will be able to find an example which corresponds exactly with the description; but in river dams and elsewhere he may find in the course of his experience quite a number of examples more or less similar which illustrate the general principle.

Briefly expressed the effect of the wall is that the water can move up and down it with little or no loss of its energy but it cannot move across it: it must confine its motion to up and down. This effect might be achieved anywhere in the deep sea without a wall if it could be arranged that the approaching train of waves be met by an exactly similar train of waves coming from the opposite direction, with the particles at the meeting-place always in the same phase of their orbit for up and down motion, and just opposite phases for the backward and forward motion. That is in effect exactly the meeting of the train of waves with its own reflexion as if the wall were a plane mirror. At the wall the vertical oscillation will be continued and indeed will be doubled in amplitude, and the horizontal component will be exactly compensated not only there but throughout the region to which the reflected wave extends.

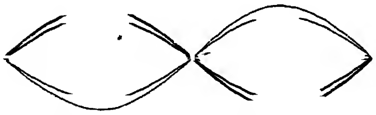


Fig. 7. Phases of a nodal wave. The successive curves, which can be identified by successive thicknesses of line, represent the positions of a string or an oscillating surface of water under the influence of a train of waves and of the perfect reflexion of the train from a vertical wall. There is a node at each end and in the middle, where there is no motion, neither vertical nor horizontal. Between the nodes there is motion in the vertical but only sufficient horizontal motion to keep the shape.

The effect in front of the reflecting wall will accordingly be a train of waves consisting of vertical motion only, with double the vertical amplitude of the incident wave at the wall and at successive half wave-lengths from it and without any horizontal motion. Nodal points where there is no motion at all are formed a quarter wave-length from the wall and at every half wave-length from the first node.

The result will be a train of "nodal waves" which pass through the phases indicated in fig. 7 without any horizontal movement either of the profile of the wave or of the water of which it is composed. The effect can be illustrated on the experimental scale by the reflexion of ripples, and with sea-waves some approximation is afforded by the combination of the reflected train with a train of waves incident upon a sea-wall; but the causes of departure from the hypothetical result are many and what appears to the spectator is a confused heaving of the water without notable progression.

The nodal waves formed by reflexion at a vertical wall are different from the standing waves described in the preceding section as formed by an obstacle in a flowing stream. The orbits of the motion of the water in the latter are vertical circles, whereas in the standing waves formed by reflexion the motion is linear.

Oblique reflexion

We have supposed the train of waves to be incident directly with its line of approach perpendicular to the wall. When the incidence is oblique, since horizontal motion is possible along the wall, it is only the component perpendicular to the wall which is compensated by a train of waves coming from the opposite side; that, combined with the motion along the wall and the vertical motion, gives a train of waves exactly like the incident train and coming from the wall at the same angle but on the other side of the normal to the face of the wall. The motion of the water in front of the wall is represented by the combination of the two trains.

Many curious results can be obtained by reflexion as the angle of incidence is changed from that of the normal to that of grazing incidence.

In the behaviour of waves which approach a wall obliquely and in the reflected waves which go off at approximately the same angle on the opposite side of the normal, the phenomena of optical reflexion can be visibly illustrated.

The effect of smaller obstructions upon wave-motion

Realising our inability to construct typical travelling waves in water to a desired pattern or size, we may understand that if we wish to use water-waves in illustration of the general effect upon wave-motion of isolated obstructions we must take the waves as we find them and pay attention to the influence of obstructions that can be found in their paths. In any case observations are not easy; most of the natural obstructions to waves of water are related to a sloping shore which has definite effects of its own. It is seldom that one gets the opportunity of watching the behaviour of a train of deep-sea waves upon a sheer cliff or other obstacle rising from great depth, and it is not easy to refer to typical examples of obstructions of a different character.

It may however be concluded from the common experience of watching the travel of waves through the structure of a pier that a single vertical pole has very little effect upon a deep-water wave approaching the shore, and a

fence of similar poles, spaced a metre apart, would afford no protection to the shore against the effect of heavy seas. Even a forest of scaffold-poles would produce very little effect upon an advancing wave; in other words, the water with a group of obstacles of that description would be almost perfectly "transparent" to water-waves.

With the multitude of radiations with which he will have to deal, the reader will do well to keep this idea of transparency in mind because transparency cannot be regarded as having a simple meaning.

A continuous wall would completely protect the water behind it so long as it lasted. It would have to bear the whole shock of the wave-energy and provide the force necessary to reverse the train: if the wall were perfectly resistant and smooth, the energy would be expressed in the "reflected" wave which would travel away from the wall to meet the succeeding waves in their advance, as we have already explained. The combination of an advancing train of waves with its reflexion is a very interesting spectacle. The conditions are not generally so regular and appropriate that the reflexion is as perfect as the image of a spectator in a mirror, which is a better example of similar action. Instead of the regular return combining with the regular advance and forming nodal waves which can be so well illustrated by experiments on strings or organ-pipes, there may be simply a terribly confused sea, full of irregularities and dangerous to everybody and everything concerned. Nevertheless, it is an example of what disturbed wave-motion can produce.

If instead of a continuous wall we have a succession of square columns (fig. 8) with gaps the same width as the columns, certainly not often to be seen as such a construction would be extremely fragile in a heavy sea, each gap would be a path by which part of the energy of the advancing wave could reach the water behind the wall. We can continue to use the image of scaffold-poles instead of columns of greater thickness if we imagine a line of poles placed so that the interval between each pair is the thickness of a pole. The idea of a column is a little better because it carries the idea of uniformity of width. One-half of the advancing wave would make its way through the intervals between the columns and then for the area of water beyond the wall the intervals would act as centres of new disturbances, regular only in their repetition. The columns themselves would act as "reflectors" and so form

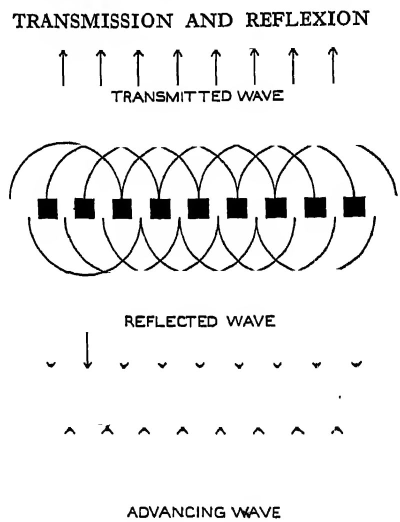


Fig. 8. "Interference." The development of waves of transmission and reflexion by a "grating" of columns, with intervening spaces of the same width. For the case of sound the actual process is shown in fig. 18.

centres for disturbance in front of the obstacles. The like is the case with light falling on a collection of obstacles. If the advancing waves are small enough to carry through the gap as formed waves, regular wave-motion will be continued on the other side. If, however, the gaps are small compared with the length of a wave, the gaps will become secondary sources of disturbance with a wave-length that is determined by the period of the disturbing oscillation and by the condition of the material which is disturbed.

We shall require the analogy of this process in the case of light. The effect, upon the water behind the wall, of the energy which comes through two adjacent apertures may be illustrated by the combination of circular waves represented in fig. 8.

The representation of a continuous wave-front as the resultant motion due to a disturbance emanating from elements of an advanced front illustrates the principle of interference upon which is based the exposition of the rectilinear propagation of waves referred to in the remarks introductory to this chapter, p. 6. This and many other illustrations of the realities of wave-motion can often be traced in water-waves by watching the behaviour of waves towards the obstacles that they may have to pass, or, using the ears instead of the eyes, in sound-waves as affected by the pales of a fence.

Diminishing waves. Damping

We have mentioned more than once the possibility of waves increasing in amplitude, in consequence of the continued operation of the forces which are responsible for their original production. Still more easy to realise is the gradual diminution of the amplitude of a wave while its period is preserved—such a diminution is known technically as the damping of the wave-motion.

To some extent the pictures of the standing waves in running water, figs. 3, 4, and 6, show what is understood by damping. Mathematicians have a device for representing the gradual change thus indicated, substituting for the constant coefficient, represented in the equation, p. 13, by A or B, one with a variable factor represented by $e^{-t/\epsilon}$ so that the full equation for a wave with a damped coefficient is

$$z = Be^{-t/\epsilon} \cos \frac{2\pi}{\lambda}(x - Vt);$$

ϵ is known as the modulus of decay, and is the time in which the amplitude of vibration is reduced to $1/e$ of its original value.

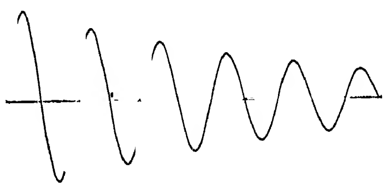


Fig. 9. A curve of damping according to the formula

$$z = Be^{-t/\epsilon} \cos \frac{2\pi}{\lambda}(x - Vt)$$

to illustrate the decay of oscillations owing to friction or other like cause.

The effect of such a term is represented in fig. 9 in which the amplitude is reduced to $4B/5$ after one oscillation. In the case represented ϵ is approximately $4\frac{1}{2}$ periods.

Curling waves and breakers

Before leaving the consideration of water-waves which we are citing for the utility of their analogy to sound-waves and light-waves we must not omit to notice the effect of a sloping bottom upon the transmission of water-waves near the shore. We have already called attention to the production of breaking waves and we now consider the deviation of the crest of the wave and the change in the direction of the motion which brings the advancing wave to face more and more nearly towards the shore.

BREAKING WAVES IN WATER

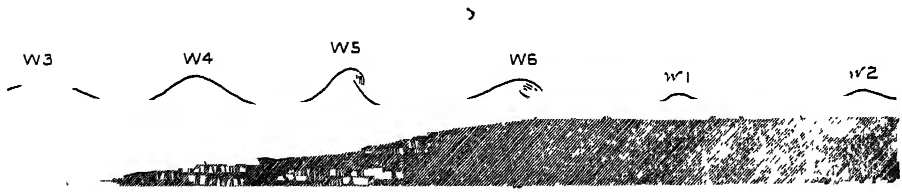


Fig. 10. Waves advancing up a shelving shore, forming a breaker or a succession of breakers, and waves in the shallow water between the breaker and the shore, with approximately the original wave-length but little elevation. (From J. Scott Russell's *Report on Waves*, B.A. Report, 1844.) For comparison with the meteorological records of a showery day see fig. 11.

The natural path of wave-motion in water, which is not otherwise disturbed, is in straight lines—to that property must be attributed the uniform widening of the circular waves which are caused by a stone dropped into a free water surface. In that case the waves are too small as a rule to show any effect of variation of depth, and indeed the law of their propagation is not that of sea-waves. The circle which limits the position of the disturbance is called the wave-front. In waves of the open sea the line of crest or wave-front is at right angles to the direction in which the waves are advancing.

The larger waves running along a shore line have their rate of travel affected by the depth, the shallower the water the slower the travel. Consequently the waves arriving at an island or projecting promontory from the distant open sea advance more slowly near the shore than farther out, the front of the wave becomes thereby distorted, the outer part curls round until it provides waves which face the shore. So it comes about that waves may be seen advancing towards the shore even when the wind is off-shore, that is to say the waves travel against the direction of the wind that was their original cause in the open sea beyond. Prof. V. Bjerknes makes use of this analogy to illustrate the transition between the wave of vertical motion which he regards as the first stage of a cyclonic depression (p. 31) and the horizontal motion of the winds round the centre of the depression.

In like manner as a wave approaches the shore (fig. 10) the motion of the lower part is retarded while the upper part retains its freedom to move, the shape of the wave is gradually altered until at last the top falls down into the front and the advancing wave carries a breaking front of foam. It is interesting

to see the foam carried up along the front of an advancing breaker. It would also be interesting to make out the closeness of the analogy which the cross-section of a wave breaking in the manner described presents to that of an eddy or vortex.

Here we make bold to introduce a picture (fig. 11) to remind the reader that there are points of analogy between the phenomena of a train of breaking waves on a shelving shore and the recurrent showers introduced by a line-squall that are often found in the south-west quadrant of a cyclonic depression

A POSSIBLE ANALOGY: BREAKING WAVES IN AIR

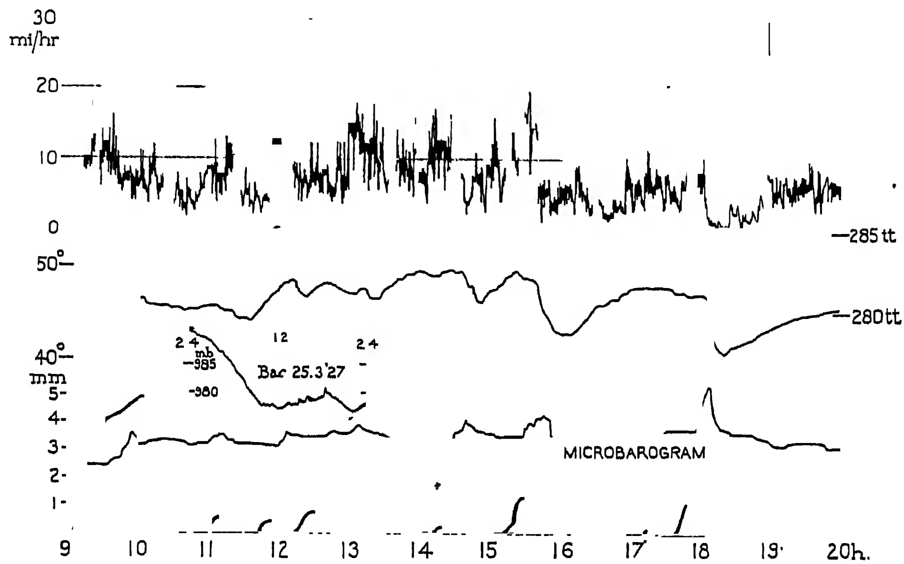


Fig. 11. The meteorological records at South Kensington of a showery day—wind-velocity, temperature, pressure, oscillations on the microbarograph and rainfall in millimetres—for comparison with the illustration of a breaking wave, see fig. 10.

during the transition from south-west winds to north-west. The analogy will not bear close examination at this stage, but in actual experience it is too marked to be disregarded; attention is called to some aspects of it in a chapter in *Forecasting Weather*[1], on the minor fluctuations of pressure.

The lowest part of the picture shows a series of nine rain showers in the course of the eleven hours included within the period of the diagram. The first was comparatively heavy. It gave rainfall to the extent of five millimetres associated as usual with a notable movement in the microbarograph. The other showers were less productive, the heaviest of them is less than two millimetres, the lightest less than a quarter of a millimetre. There are some indications of association with changes in the pressure as shown in the micro-barogram, the temperature or the wind. The whole episode is related to the gradual but irregular rise in the barogram in the corresponding period as

[1] Constable and Co., Ltd., 1923, chap. XI.

shown in the inset. Rain is itself a cause of discontinuity in the physical process. It marks and accompanies the transition from conformity with the gaseous laws to the behaviour of saturated air endowed with the energy previously latent in the water-vapour. The dynamical results have yet to be explored. Moreover the record of rain takes no account of that formed into drops but evaporated on the way down.

GRAVITY-WAVES IN AIR

Water-waves represent in effect the apparent travel of a heap of water in shape more or less like the bulge of a sine-curve followed by a corresponding hollow. We have noted that the water of the heap and the hollow does not travel with the wave but the shape does. We have learned also that the wave-motion is not confined to a single surface-layer but is found below the surface as a wave of which the amplitude diminishes as the depth increases according to the ordinary logarithmic law: that is to say, if the amplitude of the wave at 10 metres below the surface is nine-tenths of the amplitude at the surface the amplitude at 20 metres depth will be 81/100ths and at 100 metres about 35/100ths.

There is every reason for supposing that a similar action might take place in the atmosphere, the amplitude diminishing upward (perhaps downward also) from the wave-surface, if there were facilities for accommodating the heaps and hollows of the wave of air such as are allowed to water by the freedom of the space above it.

If such waves exist we should expect to find them recorded as periodic variations of pressure in a barogram, possibly associated with variations of the same period in the motion of the air as recorded in an anemogram. To get these variations properly exhibited the recording instruments ought to be kept fixed in position; and unfortunately that condition practically limits the records to those of fixed observatories. It is hardly to be expected that the associated changes of temperature would be large enough to show in a thermograph; but it cannot be regarded as improbable that if the motion is continuous and slow, the process of heaping up of the air in a wave should lift the upper part sufficiently to reduce its temperature below its dew-point and consequently to show the crest of a wave by the formation of a cloud, or by the increased density of cloud in a continuous layer.

Records in wave-form are found occasionally in barograms; we have cited a good example as illustrating the embroidery of the barogram in chap. IX, p. 391, of volume II. Another example is shown in *Forecasting Weather*, 2nd edition, p. 355—oscillations of pressure at Eskdalemuir on 6–7 March 1918, constituting two well-formed complete waves of forty minutes' period, were developed from less well-formed waves of about sixty minutes' period, and lapsed into irregular fluctuation without any recognisable period. The variations shown in pressure have also their counterpart in variations of the same period in the direction and force of the wind.

The forty minutes and the hour of that example are less usual than the twenty-minute or ten-minute oscillation of fig. 216 of vol. II, and indeed the frequency of ten-minute or twenty-minute oscillations is suggestive of some prevalent natural cause of waves in the atmosphere. D. Brunt[1] has sought to relate such a period to the prevalent lapse-rate of the atmosphere by computing the time of oscillation of a specimen of air moving in its environment as a material particle; he obtains a time of oscillation dependent upon the lapse-rate of the environment varying between 6 minutes for the isothermal condition and an infinite period for convective equilibrium, with an intermediate value of 10 minutes for a lapse-rate two-thirds the adiabatic; for periods 20, 30, 40 minutes the corresponding lapse-rates are ·92, ·96, and ·98 of the adiabatic. The calculation does not express accepted hydrodynamical principles and methods; but the result invites further effort to obtain a solution of the problem.

The variability of the oscillations is well illustrated by fig. 12 which shows the record of the anemograph of the Meteorological Office station at Fleetwood, on 4–5 February 1927. It will be seen that from 17h to 19h the waves are almost exactly of a half-hour period. Between 20h and 22h six maxima are included and thereafter the orderly oscillation is at an end though there is an indication between 1h and 2h of an oscillation of 15-minute period better shown in the direction trace than in that of the velocity.

Oscillations very similar to those represented in fig. 12 have been previously observed at Southport and are in some way characteristic of the eastern shore of the Irish Sea, with its mountainous amphitheatre. The subject is discussed in a paper before the Royal Meteorological Society in 1910 (*Quarterly Journal*, vol. XXXVI, p. 25).

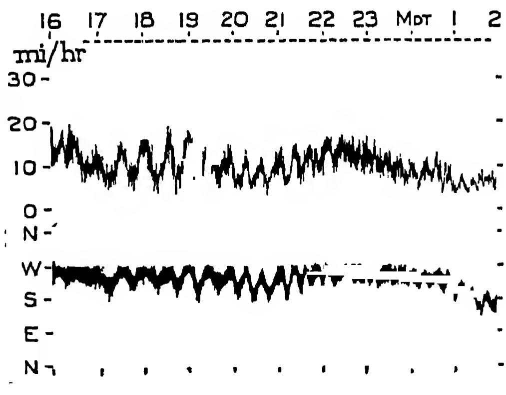


Fig. 12. Oscillations in the velocity and direction of the wind shown by the pressure-tube anemometer at Fleetwood, 4–5 February 1927.

The striped appearance of cloud in the sky may also be an indication of wave-motion marked by condensation in the ridges. It is quite a common occurrence and is represented by many examples among the photographs of volume I. It is natural to suppose that the lines of corrugation mark the lines of crests and hollows of waves in the atmosphere which are travelling across the lines, though another explanation of the corrugated appearance, which will be cited in chap. VIII, is offered by T. Terada and others.

Whatever may be the difficulty of working out the analogy between pro-

[1] *Q.J. Roy. Meteor. Soc.* vol. LIII, 1927, p. 30. The subject has been further examined by N. K. Johnson, see *Ibid.* vol. LV, 1929, p. 19.

gressive waves in water and in air there can be no doubt in respect of the analogy between the standing waves in a stream of water which flows past an obstacle as represented in figs. 3–6 and waves in the layers of the atmosphere in which the conditions are quite similar. The study of the motion of air past obstacles is indeed fundamental for many problems of aircraft. On the large scale of the natural obstruction of orographic features to the flow of air over or round them which was discussed by V. Bjerknes[1] the same phenomena are in evidence. They may account for the lenticular forms of cloud which are represented in chap. XI of vol. I. In particular the *baleia* of Mt Pico in the Azores, fig. 48, and the *Contessa del vento* of Mt Etna, fig. 46, may be referred to as clouds which are permanent in position and constantly replenished by the wind[2].

Waves in an air-current as persistent and well-shaped as those represented for water by J. Scott Russell (fig. 6) would presumably require the air of which they are formed to be in permanent rotation as vortices with horizontal axes, not to suffer any translation but to have its radius of operation adjusted to keep the profile of the wave. That is not quite the case in practice, as imperfect vortices are often transmitted down stream (Part IV, chap. V, fig. 7) and the like occurs with the wind if we may thus interpret the transitory variations of an anemometer.

Information has recently come to hand that certain oscillations of pressure recorded on 30 June 1908 by microbarographs in England with periods of ten and twenty minutes (*Forecasting Weather*, 1923, p. 356) may be attributed to the fall of a large meteorite in northern Siberia[3], but the genesis of gravity-waves in air is not generally understood. As a guide to the conditions necessary for their development we must refer to the lines of equal entropy in the atmosphere and the degree of closeness of their approach (fig. 63 of vol. II). They mark the surfaces which cannot be crossed by a sample of air at the boundary without causing gravitational resilience of the same kind as that which affects a boat when it is temporarily pushed down below its natural water-level, or lifted above it. The closer the approach of the lines of equal entropy the greater the resilience for the same amount of displacement.

The level of the sea itself is the most conspicuous example of a resilient surface for air because there is a discontinuous jump from the density of the water to the density of air, and air submerged in the water is subject to a force of resilience equal to 800 times its own weight. The gradations of entropy in the atmosphere do not reach actual discontinuity; but in any layer which represents a counterlapse (inversion of lapse-rate) there is a rapid increase of entropy and the condition of discontinuity is more or less nearly approached.

For example, a stream of relatively warm air passing over a level and very cold surface-layer (such as those represented in chap. V) may operate upon

[1] 'The structure of the atmosphere when rain is falling,' *Q.J. Roy. Meteor. Soc.* vol. XLVI, 1920, p. 119.
[2] C. K. M. Douglas, 'Some Alpine cloud-forms,' *Q.J. Roy. Meteor. Soc.* vol. LIV, 1928, p. 175.
[3] *Bull. Amer. Meteor. Soc.* vol. IX, 1928, p. 213.

the cold layer beneath it much in the same way as air acts upon water, and, when there are two sides to the layer of rapidly varying entropy, waves may be set up in the upper layer or the lower layer or in both. We defer the consideration of the dynamics of the problem until we have set out the general equations of motion of the atmosphere as an introduction to vol. IV, merely mentioning that on the basis of a discontinuity between equatorial and polar air, which provides the keynote of the new theory connoted by the polar front, Helmholtz has deduced equations for wave-motion in the region of the discontinuity with special reference to waves indicated by the

GRAVITY-WAVES IN AIR

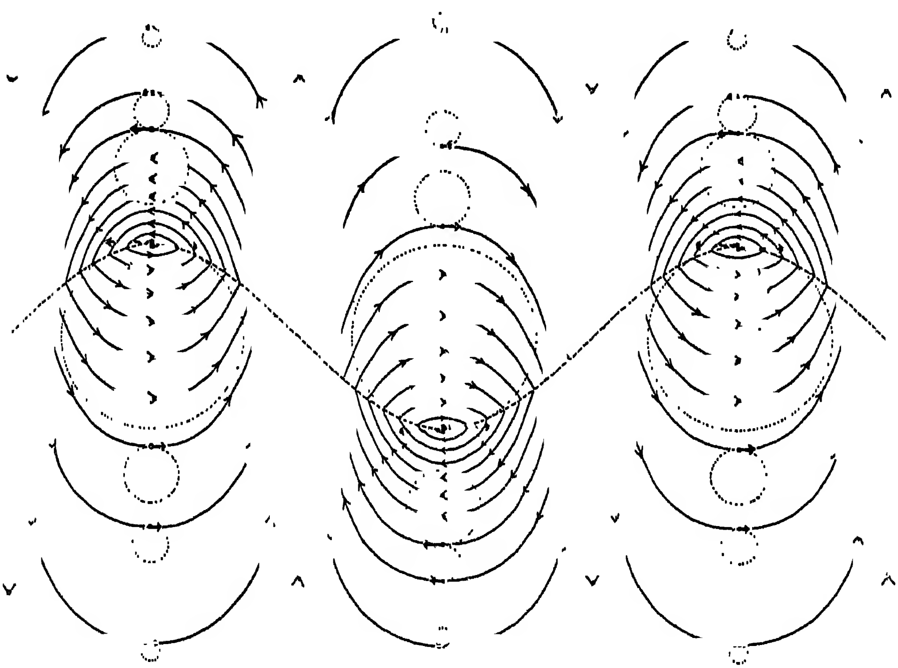


Fig. 13. Gravity-waves above and below a surface of separation between two fluids of different density and in motion relative each to the other. (V. Bjerknes, 'On the Dynamics of the Circular Vortex,' *Geofysiske Publikationer*, vol. II, No. 4, Kristiania, 1921.)

clouds; the theory has been developed by Brillouin as well as by Wien and F. M. Exner. Another attempt at the general theory of wave-motion in the atmosphere is that of Horace Lamb[1], which was written in reply to an appeal for an explanation of the periodic variations of pressure that are occasionally recorded.

V. Bjerknes, in developing the theory of the polar front, has expounded the development, on either side of a discontinuity, of waves which travel with the air of the layer which has the greater velocity. A sketch of the waves thus indicated is given in fig. 13. From the theoretical result Bjerknes would explain the formation of cyclones in temperate latitudes.

[1] 'On atmospheric oscillations,' *Proc. Roy. Soc.* A, vol. LXXXIV, 1911, p. 551.

Other atmospheric waves

The structure of waves excited a good deal of attention among those who were interested in physical science in the first half of the nineteenth century and efforts were accordingly made to represent the travel of pressure, expressed by a barogram, as due to the motion of linear waves of pressure over the country. The idea still finds favour in the north of Italy where cyclonic and anticyclonic distributions are not nearly so apparent as they are in the weather-charts of northern Europe.

Within the past ten years L. Weickmann of the Geophysical Institute of Leipzig has suggested a special form of wave-progression over northern Europe which is derived from the weather-charts themselves.

In a lecture before the Deutsche Meteorologische Gesellschaft in October 1926 Weickmann gives an effective summary of Exner's attempt to synthesise the variations of pressure from solar radiation and the distribution of land and water, together with the efforts on the part of others, especially of H. H. Clayton, Defant, Danilow, Matteuzzi and Vercelli, to detect periods in the recorded variations of pressure in different parts of the world.

The summary is the preface to an account of work carried out by himself and others at the Geophysical Institute at Leipzig which has enabled him to represent the variations of pressure over the northern hemisphere during selected spells of ten weeks' duration, more or less, as made up of the oscillation of nodal waves. The spells are chosen by the identification of points of symmetry in the graph of pressure, and the pressure curve for the spell is analysed into sine components. Particulars are given for the spell of 72 days, 36 on either side of 15 January 1924 as the point of symmetry. By analysing curves for 800 stations in the northern hemisphere the amplitude and phase of a standing circumpolar wave of 24 days' period were determined which expressed the pulsation of the polar front for that period of the year.

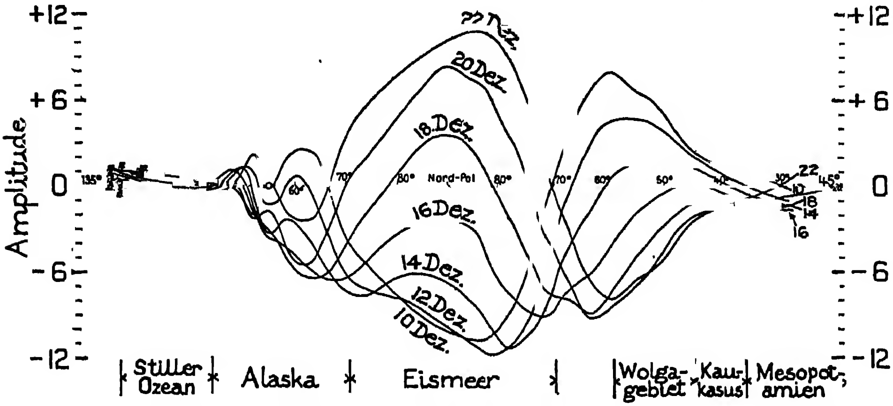
Den Schnitt längs des 45 Meridians westl. Länge in Abständen von zwei zu zwei Tagen zeigt Fig. 14. Er führt im wesentlichen vom Atlantischen Ozean über das Polarmeer zum Pazifischen Ozean und gibt somit die Verhältnisse ohne Störung durch die kontinentalen Einflüsse wieder. Die Kurven umfassen eine halbe Wellenperiode, also 12 Tage, die folgenden Kurven für den 22, 24 usw. Dezember oder die homologen um $2n\pi$ verschobenen Termine sind die Spiegelbilder der ersten sechs. Man sieht in eindrucksvollster Weise eine 24-tägige Pulsation der Polarluftmassen vor sich, keine Erscheinung des kontinental-maritimen Systems, sondern der Polarfront, also nicht von Westen nach Osten sich fortpflanzende Wellen, wie sie aus den Defant-Exnerschen Arbeiten bekannt sind, sondern meridional verlaufende Schwingungen, die sich mehr mit den Wahrnehmungen von Danilow decken. Natürlich hat man sich dieses "System" superponiert zu denken auf die mittlere Luftdruckverteilung, deren Translationsbewegung es unterworfen ist, und die den wohlbekannten Charakter einer winterlichen Luftdruckverteilung aufweist für einen mittelstrengen Winter, wie es der Winter 1923/24 gewesen ist, also Hochdruckgebiete über den Kontinenten, Minima über den Ozeanen.

(L. Weickmann, 'Das Wellenproblem der Atmosphäre,' *Meteor. Zeitschr.* 1927, p. 250.)

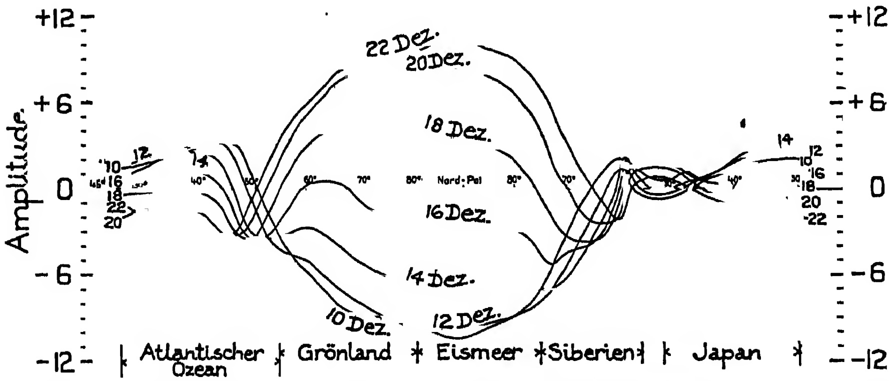
Another nodal wave of 36 days' period between centres distributed in longitude instead of latitude is suggested as showing the effect of the distribution of land and water.

It is not claimed that these oscillations are persistent throughout the year nor from year to year at the same season; separate spells have to be chosen

Fig. 14. Oscillations of pressure in the winter of 1923–24 suggested by a nodal point of symmetry in the barograms of the northern hemisphere (Weickmann).
The sequence of one-half period of the 24-day wave in sections along meridians.



I. Curves of departure from mean pressure in a section of the surface-air along the meridian of 135° W 45° E, at intervals of two days.



II. Curves of departure from mean pressure in a section of the surface-air along the meridian of 45° W 135° E, at intervals of two days.

for successive seasons, but, once selected, they may be relied upon to exhibit the special character of the particular season to which they refer.

The conclusions arrived at with regard to a nodal oscillation of 20 days' period in summer with a node in Russia and correlated maxima in the northern North Sea and south Caspian, are summarised by Weickmann as follows:

Zusammenfassung: Die harmonische Analyse der Luftdruckkurven von ca. 300 Stationen von Europe, Westasien und Nordafrika für die Zeit vom 15 April bis 3 Juli

1925 ergibt unter anderem eine 20 tägige Welle, die Amplitudenmaxima über dem Nordmeere und über Asien aufweist. Sie hat den Charakter einer stehenden Welle; die Amplitudenmaxima zeigen alternierende Pulsation. Ein Zusammenhang dieser Welle mit dem europäischen Monsun ist wahrscheinlich. Solche Wellen schaffen Bezugsräume des Luftdruckganges. Durch die üblichen Korrelationsrechnungen kann daher nur ein Teil der Erscheinungen erfasst werden. Die Wellen wirken als Impulse in der allgemeinen Zirkulation der Atmosphäre.

('Die Ausbreitung von Luftdruckwellen über Europa,' *Gerlands Beiträge zur Geophysik*, Bd. XVII, Heft 3, 1927, p. 332.)

Diurnal waves of pressure

We have referred to the barogram as the material in which to seek the evidence of wave-motion in the atmosphere due to gravitation. Any reader who acts upon the suggestion is very likely to light upon the semi-diurnal wave of pressure which travels round the earth about two hours in advance of the sun (vol. II, p. 281) as being the most easily recognised example, especially in latitudes nearer the equator than 35°. One can hardly fail to see that, and one may fail to see any other evidence of waves. We have already described the distribution of these waves in the volume referred to. The semi-diurnal wave, however, as investigated by Lord Rayleigh and subsequently by Margules is not a gravity-wave in the sense in which we have been using the term, but a wave that depends upon the elasticity of the air. The resilience which calls forth waves depends upon the compression and rarefaction of the air in the line of motion instead of the increase or decrease of weight.

At every station where barograms are kept a diurnal variation of pressure with a period of 24 hours can be demonstrated by taking the mean values of the pressure at each of the 24 hours of the day. In that general oscillation must also be included the 12-hour term, and it is usual to ascertain the particulars of the 12-hour term by regarding it as the second component of the harmonic analysis of the variation within the 24 hours. According to Margules's theory the 24-hour term, which exhibits little sympathy either in amplitude or phase between different localities, does not depend upon the elasticity of air. If that be so, it must depend upon elements like temperature, the variations of which are clearly dependent upon the alternations of day and night; and it is too much to hope that those influences will be so perfectly expressed by a pure sine-curve of 24 hours' period that they will be eliminated in the process of harmonic analysis. Its influence may therefore affect the purity of the expression of the 12-hour components and its harmonics at different stations.

CHAPTER II

SOUND-WAVES

It is one of the most important principles in connection with the transmission of energy by waves that we have to distinguish between the velocity of the waves and the velocity of the energy they are carrying, the greater the velocity of the waves the smaller is that of the energy This fundamental principle is apt to be overlooked, for, in the most conspicuous cases of wave-motion, sound and light, all the waves travel with the same velocity, so that the question of the alteration in the speed of energy does not arise

(J J Thomson, *Beyond the Electron*, 1928)

We have already remarked that the semi-diurnal wave of pressure, the best known of all atmospheric waves, owes its travel to the compression and elasticity of the atmosphere. Waves of sound are of a similar character. They are intermediate in length between the water-waves which are long and light-waves which are extremely short. They differ from water-waves also in the fact that the water-waves are made up of particles which describe orbits in planes at right angles to the wave-fronts, and therefore require some time at any rate for their development, or perhaps it may be called their "education" from a state of rest, whereas the particles which form sound-waves move simply to and fro along the line of motion, they require no time for their education, only for transmission. A single explosion produces a group of sound-waves in response to the impulsive compression of the air surrounding the exploding mass, and the rate of travel is in accordance with the law of transmission of sound-waves, $V = \sqrt{\gamma p/\rho} = \sqrt{\gamma R tt}$, where V is the velocity of travel, p the pressure of the air, ρ its density, tt its temperature, γ the well-known ratio of the two specific heats of air, and R the "gas-constant." So clearly is this recognised that the "velocity of sound," about 330 m/sec, is one of the common items of reference in experimental physics.

The velocity of sound has been measured by direct observations in the open air. Classical experiments for this purpose were conducted by Arago in June 1822 between Montlhéry and Villejuif.

After Laplace had pointed out the source of error in Newton's calculations, a Commission was appointed by the Bureau des Longitudes which experimented again on the outskirts of Paris, between Montlhéry and Villejuif, a distance of about 11 miles. Reciprocal cannon-firing was used, but at intervals of only five minutes, and chronometers replaced the pendulum clocks of the first experiment. At 15·9° C [288 9tt] the result was U = 340 9 m/sec, whence U_0 = 331 m/sec.

(J. H. Poynting and J. J. Thomson, *Sound*. Charles Griffin and Co. Ltd., London, 1899, pp. 24–5.)

The most effective determination is by means of nodal waves which in the case of water we have mentioned as being imperfectly developed by the reflexion of waves from a vertical wall. The lengths of such nodal waves in air are beautifully marked by light powder in a glass tube along which a sound is transmitted. The interval between the lines of powder gives the half wave-length, the pitch of the note the period, and, from the ratio of the two, the velocity of transmission can be accurately determined.

The value of the velocity of sound in air at 273tt as determined by this method is 331·90 metres per second.

The method, which is due originally to Kundt and Warburg, was used extensively by J. W. Capstick[1], who applied it to the determination of the ratios of the specific heats of various gases.

With this comparison in view we need not regard the velocity of a group of sound-waves as something different from the theoretical velocity of sound, the difference will not trouble the meteorological observer unless we must attribute the curious limitation of the audibility of thunder to a cause of that kind.

The energy which is expressed in sound is not itself of sufficient importance to claim our attention. R. L. Jones has calculated that a million persons would have to talk steadily for an hour and a half to produce enough heat to make a cup of tea[2]. Sound, however, furnishes a remarkably sensitive divining rod for the structure of the atmosphere, because the travel of sound is strictly dependent on the condition of the atmosphere which is traversed by it.

The travel of sound, the basis of "sound-ranging," can indeed be used to identify the position of a distant explosion by noting the time of the flash and the time of arrival of the report at two or more stations equipped with instruments for recording the reception of sound. The method has been so far developed as to require scrupulous attention to the influences of all the various meteorological conditions upon the travel of sound[3]. It has not yet been brought into use for the benefit of the science of meteorology to anything like the extent which seemed possible at the close of the war.

All sounds travel with the same velocity in the same air, though their wave-lengths may vary from 10 metres for the hardly audible note of an organ-pipe, to less than 1 cm for the highest audible note of an adjustable whistle devised by Galton. The limit in frequency is from about 30 oscillations per second to 24,500 per second. All sound-waves can be reflected from plane surfaces and have the faculty of creeping along curved surfaces by repeated reflexion as in a whispering gallery; but there is great difference in the treatment which the waves of different length receive from obstacles. The long waves get much disturbed by partial or complete reflexion. Reflexion from obstacles and diffraction from the broken edge of a sound-wave are so effective that sounds of moderate wave-length have no noticeable shadows; they can make their way, presumably by sacrificing some of their energy of compression, through walls and windows and are audible in situations where light from the same source would be invisible. It is difficult to reconcile the experience with rectilinear transmission. But the realities of ordinary wave-motion are exhibited very clearly with the smaller wave-lengths such as those of Galton's whistle or the common bird-call. A great number of interesting phenomena in audition are really dependent upon sound-shadows and can be disentangled

[1] *Phil. Trans. Roy. Soc.* A, vol. CLXXXV, part I, 1894, p. 1.
[2] *Nature*, vol. CXXI, 1928, p. 612.
[3] See W. J. Humphreys, *Physics of the Air*, 2nd ed. 1929, p. 418.

by careful observations; but we have not space for details on that subject. It is upon such details that the acoustical properties of buildings depend, and one of the signs of revival of interest in the physical problems of fifty years ago is the attention which is now devoted to practical acoustics[1].

VARIATION OF WAVE-FRONT DURING TRANSMISSION

For our purpose it is better to regard the transmission of a sound rather as the advance of a wave-front than as the direct operation of radiation in straight lines. On the analogy of the waves which spread out from a centre of disturbance in water, a disturbance representing sound coming from a point in a perfectly homogeneous atmosphere would be a spherical surface advancing outwards in all directions with "the velocity of sound." The wave-front will be spherical and the travel will be at right angles to the front. At a great distance from a source the front is practically plane and the disturbance advances at right angles to the plane. So we may consider a sound-wave as having a spherical front or a plane front or a distorted front according to the circumstances.

The natural atmosphere is never perfectly homogeneous and the spherical wave in the open air is an unrealised abstraction because the velocity of transmission is affected by the temperature, by the wind-velocity and, near the ground, by the obstacles which have to be circumvented.

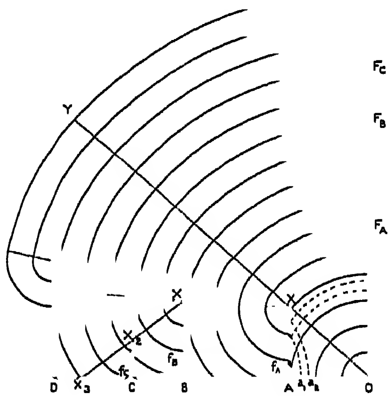


Fig. 15. Wave-fronts (originally spherical) for transmission of sound past a series of obstacles.

O, centre of disturbance with wave-front originally spherical.

A, B, C, D, obstacles in the way of the advancing wave-fronts.

$F_A f_A$, $F_B f_B$, $F_C f_C$, wave-fronts distorted by diffraction at the obstacles A, B, C.

XY, XX_1, X_1X_2, X_2X_3, the "rays" which bound the regions that can only be reached by diffraction.

$a_1 a_2$, wave-fronts of reflexion from the plane face of the obstacle A.

We will begin with the consideration of the interference with transmission due to the obstacles, or the "friction," of the surface which has the effect of diminishing the velocity of the waves near the ground. We may assume for the time being uniformity of temperature and no wind. The problem that we have to consider is such as that of the explosion which occurred at Silvertown on the Thames estuary in 1917. It rattled windows and even pushed open a door in a basement eleven miles to the west, separated from the source of the disturbance by miles of earth and buildings. A corresponding effect must have been produced all the way along the route. The energy displayed at street-level, or

[1] See for example *The Acoustics of Buildings* by Dr A. H. Davis and Dr G. W. C. Kaye, London, 1927; 'Quietness in City Offices,' *The Times*, 5 Feb. 1929; 'The New Acoustics' by Dr W. H. Eccles, Presidential Address to the Physical Society in 1929.

below it, miles away from the source, came from the explosion not by transmission in direct lines but by what is called diffraction from the broken edge of the wave-front, that is to say, as the wave passed each obstacle the excess pressure at the lower edge could not be retained in the absence of suitable support; the overhead energy that survived would act as a source sending down disturbance so that the wave-front would become bent as in fig. 15. The line of travel of energy which reached the ground twenty miles away would have started from the source as a ray inclined at a considerable angle. It reached the ground, farther on, by curling round the obstacles, thus travelling a longer path; and therefore its progress "along the ground" would be slower than the velocity of sound.

Reflexion

We can deal in like manner with reflexion of sound-waves. The case of reflexion from solid walls and ceilings, by which echoes are produced, is simple enough. With a perfectly rigid plane surface no energy is lost, the wave

REFLEXION AND REFRACTION OF SOUND-WAVES

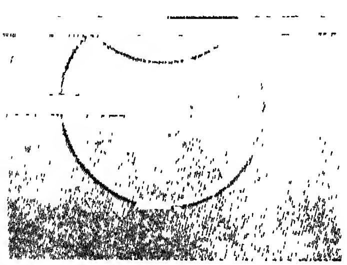


Fig. 16. "A wave reflected at the plane surface of a piece of glass plate about 10 cm wide and 20 cm long, held vertically a short distance from the spark gap, with the surface of the plate parallel with the gap." (A. L. Foley and W. H. Souder, 'A new method of photographing sound-waves,' *The Physical Review*, vol. XXXV, 1912, Photograph 11.)

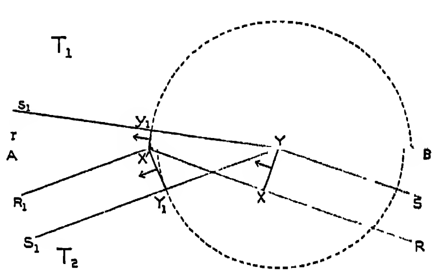


Fig. 17. Hypothetical reflexion and refraction of a plane wave of sound at the surface of separation AB of two homogeneous media of temperatures T_2 and T_1 of which T_1 is the higher. XY is the incident wave-front, $X'Y_1$ the reflected wave-front and $X'y_1$ the refracted wave-front. The dotted circles, drawn to scale, represent the fronts of the waves of reflexion and transmission from the element Y.

proceeds from the reflecting surface backwards as though it came from the "image" of the source formed in the same way as by optical reflexion. Partial reflexion can take place from any layer of air which is separated from the source by a surface of discontinuity of temperature or wind.

The reflexion of a circular wave from a rigid linear boundary is very clearly shown in the photographic picture represented in fig. 16. The appearance like water in the photograph is the optical record of the distortion of the uniform atmospheric condition caused by the condensation-wave of sound sent out

from an electric spark under the spherical knob which is shown black. The reflecting surface is shown by the horizontal black band.

The supplementary arc, which would complete the circle of the wave-front if it were reversed, is the front of the wave reflected from the wall.

Ordinary echoes which are explained on the principle here illustrated are produced by reflexion from vertical walls of rock or other natural objects, but they have not the regularity of that shown in the figure.

In the medium in which an incident wave-front is travelling reflexion may be represented as the integrated effect of disturbance by spherical waves emanating from those parts of the reflecting surface that are successively affected by the incident wave. On the principle of rectilinear propagation (p. 6), a plane wave-front XY (fig. 17) which impinges upon a plane surface AB may be regarded as exciting disturbance along YX′, the part of the surface contained between the two rays R and S drawn at right angles to the front. The disturbance begins at Y and reaches X′ after the distance XX′ has been traversed. In the meantime the disturbance originating from Y will have spread out into a spherical wave of which the section is the circle passing through Y_1 with centre Y and radius YY_1. Since only one homogeneous medium is concerned in the transmission, the velocity of travel will be the same as for the incident plane wave. The elementary spherical wave, YY_1, will therefore be equal to XX′.

Corresponding waves will be sent out from all the successive points along YX′, and the radii will all be smaller than YY_1, ultimately being zero at X′, and all the waves will be touched by the line $X'Y_1$ which may therefore be regarded as a wave-front advancing parallel to itself and bounded by the rays Y_1S_1, $X'R_1$ inclined to the reflecting surface at the same angle as the incident rays.

In cases of perfect reflexion the wave-front may indeed be regarded as integrating the whole energy initiated by the disturbance emanating from the reflecting surface YX′; the elements of hypothetical disturbance of the other parts of the medium compensate each other by interference. The intensity of the energy necessary to achieve the reflexion is gauged by the exact reversal of the motion at right angles to the reflecting face.

Refraction

Perfect reflexion as represented in figs. 16 and 17 may occur when the surface upon which the sound is incident is rigid, and the elasticity of the moving air is perfect. Both these requirements are quite sufficiently well represented by a solid plane wall and ordinary air. But there are many interesting cases of the transmission of sound through the atmosphere when there is a change of transmission in consequence of change of temperature. Where there is discontinuity of temperature, partial reflexion will take place, the greater part of the energy being devoted to forming a wave in the new medium or in the old according to the angle of incidence.

For the purpose of illustration we will suppose that the two media are separated by a plane surface and that each is homogeneous, which for the purpose of transmission of sound means that it is of uniform temperature, the one T_2 and the other T_1 of which we will suppose T_1 to be the higher.

In this case only part of the energy of the disturbance caused at Y (and other points along YX′) by the incident wave XY in the medium T_2 will find its integral in the wave-front $X'Y_1$ in the same medium, the remainder will excite disturbance in the medium T_1 on the other side of the surface. But here the velocity will be different as it depends on the temperature. In the case which we have represented the velocity will be larger in T_1 because the temperature of T_1 is higher.

In fig. 17 if the change of temperature at the boundary be assumed for purposes of illustration to be 30tt, from 273tt to 303tt (an assumption which can only be justified in exceptional circumstances) the velocities in the two media will be 332 m/sec and 350 m/sec. The transmission of the sound in the warmer medium will be represented by the spreading out in T_1 of the disturbance from Y over a spherical front with radius Yy_1 which bears to YY_1 the ratio 350/332, i.e. 1·055. The diagram is drawn to scale and the wave-front which integrates the disturbance of the second medium T_1 is represented by $X'y_1$, the tangent drawn from X′ to the circle centre Y and radius 1·055 YY_1. The portion of the new wave-front excited by the disturbance incident upon X′Y is bounded by the rays Ys_1, $X'r_1$ and these are inclined to the normal to the surface at an angle j related to the angle i of incidence by the equation $350 \sin i = 332 \sin j$.

Thus if the angle which the incident ray makes with the normal to the surface of separation is 70° the refracted ray is deviated through 12° from the incident ray; or expressing the same facts in another way, the wave-front is turned through an angle of 12° counter-clockwise, and is more nearly vertical by 12° than the incident front. A vertical front implies horizontal motion, so the rays (which mark the direction of motion) are by 12° more nearly horizontal in the medium T_1 than in T_2.

Diffraction

We have based the explanation of the phenomena of transmission of sound on the hypothesis of the integration of disturbances emanating from different parts of an advancing wave-front. If the reader wishes to convince himself of the scientific propriety of that hypothesis he cannot do better than try an experiment which the late Lord Rayleigh used in order to demonstrate the acoustic analogy of the optical experiment of "Huyghens's zones." It may be recalled that when Fresnel presented to the French Academy his undulatory theory, which relies upon the hypothesis of the interference of vibrations, Poisson pointed out that it would follow from the theory that there should be a bright spot at the centre of a shadow of a circular obstacle thrown on a screen by a luminous point; and on trial that proved to be the case. Huyghens's

zones consist of a central circular disc to form the shadow, and concentric rings surrounding it, each of the same area as the central circle with intervening circular spaces also of the same area. Such a series of zones concentrate the light from a luminous point like a lens. The experiment works perfectly also with sound, the zones being on a larger scale than those used for light. They can be cut out of any opaque material.

Shadow phenomena with sound are sharper for smaller wave-lengths; the experiment with Huyghens's zones is most effectively shown by the high note of a whistle or a "bird-call" or an electric whistle and a sensitive flame; but anyone who likes to try it on the larger scale of waves of a metre length may safely be promised an effective demonstration.

It must be remembered however that the zones have a focus like a lens, because the condition of concurrent reinforcement of the disturbance, from corresponding rings of all the zones, at a single central point is that the difference of path to the centre of the shadow from one ring of opacity and from the next should be a wave-length or an exact multiple of it.

The diffraction of a sound-wave is quite similar to the diffraction of a water-wave by a screen represented in fig. 8. A series of parallel openings forming a grating for sound-waves is exhibited in the photograph of fig. 18 which was obtained by the same procedure of successive electric sparks as that employed for fig. 16. The secondary wave-fronts of reflexion from the bars and of transmission through the openings are clearly shown.

Fig. 18. A sound-wave reflected from and transmitted by a diffraction grating; both the reflected and transmitted systems of waves are in complete accord with Huyghens's principle. (Foley and Souder, *loc. cit.* fig. 16, Photograph 25.)

"The grating was made by cutting four equal and equally spaced rectangular slits in a strip of sheet tin. The slits were 7 mm wide and 35 mm long, with a strip of tin 7 mm wide between the openings. The tin was tacked to a wooden block which served as a supporting base. The grating is placed with its apertures parallel with the spark gap." In the photograph the positions of the slits are shown by a thinner shadow than the adjacent reflecting surfaces.

WAVE-FRONTS IN THE ATMOSPHERE

In representing the reflexion and refraction of sound on the hypothesis of wave-motion we have supposed we might deal with a plane wave-front. But in the atmosphere sounds have generally to be considered as coming from a point as source, or something like it, and as spreading out originally in spherical waves. An actual wave-front may become approximately a plane wave-front at a distance from the source, either by the gradual increase of the radius or by some effect of the varying velocity upon the shape of the front.

It will therefore be of interest to consider the transmission of sound rather as the life-history of a wave-front originally spherical than as the operation of radiation in straight lines, understanding that the velocity of transmission at any part of the front is at right angles to the front and depends on the temperature of the air.

The effect of temperature

Let us suppose a source of sound such as a fog-horn near the surface of the sea. After one second from the start the front may be represented by a hemisphere. We may also suppose that the temperature is falling off rapidly with height, but is not appreciably altered in any horizontal direction. The part of the front in the zenith will accordingly move more slowly than along the horizon, and when the wave-front reaches the stratosphere it will have become flattened (fig. 19). From that position, if the stratosphere is isothermal, the upward velocity will not change.

Fig. 19. Refraction of sound. Wave-fronts of a sound-wave, originally spherical, in an atmosphere in which the temperature falls off with height, but is uniform in the horizontal The diagram is drawn approximately to scale for a change of temperature from 300tt at the surface to 230tt at 10 km

The width of the diagram is 200 km and the height 10 km, the interval between successive wave-fronts is 30 seconds

With an approximately uniform lapse-rate between the stratosphere and the surface, the lower layers, with the exception of that which is affected by the ground, will travel faster than the higher, and the wave-front will tend more and more to face the sky and the sound-ray to leave the earth. Hence we may conclude that elevation of the receiving station improves the hearing of a distant sound. That may afford an explanation of the better hearing of surface sounds in the car of a balloon than the hearing at ground-level of a sound originating at the balloon

The effect of wind

If the air in which a sound-wave is travelling is itself in motion, the velocity of travel of the sound will be increased by the velocity-component of the wind in the direction of motion of the sound, and correspondingly diminished by the oppositely directed component. Similarly the sound will be refracted by a cross-wind.

If we confine our attention to the travel of the sound in the line of the wind we may think of the nearly vertical sides of a spherical wave round the point H in a wind represented in direction by the arrow (fig. 20). If we may suppose the velocity of the wind to increase continuously with height the spherical shape will be distorted and both the fronts will be turned counter-clockwise. To an observer at A, up-wind, the deviation of the front will cause that part of it to move upward and be lost. At B, down-wind, the front will be bent

downward and part of the energy of the front, which would otherwise have gone overhead, will come down to the ground. It follows that at the surface sound will be stronger and remain audible at a greater distance down-wind

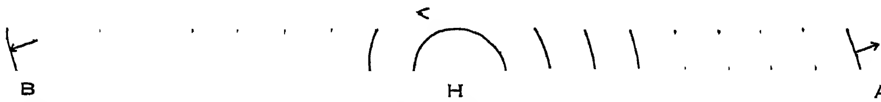


Fig. 20. Refraction of sound by wind. Wave-fronts of a sound-wave, originally spherical, in an atmosphere in which the wind increases rapidly with height. In the diagram the direction of the wind is represented by the arrow from right to left. The velocity is assumed to vary from 3 m/sec at the surface to 22 m/sec at 1 km. At A facing the wind the wave-front is bent upwards; at B with its back to the wind the front is bent downwards.

The width of the diagram is 20 km and its height 1 km; the interval between successive wave-fronts is 3 seconds.

than up-wind. This is certainly the case for the layers nearest the surface in which the increase of velocity with height is very notable.

These changes can be included in the general term of the refraction of sound by wind.

Transmission in an irregular atmosphere

Near the surface the sound-wave will be constantly frittered away by the surface-obstruction in the manner which we have described on p. 37 and the energy of the upper part will be utilised to supply sound to more distant places.

If the atmosphere is made up of a mixture of pockets of cold and warm air in juxtaposition the maintenance of a regular front will not be possible and sound will in consequence be weakened. All these conditions may occur in the case of thunder when the atmosphere is notoriously complex in its structure.

Thunder has been recognised as audible two minutes after the lightning, which would imply a distance of 40 kilometres, but that is quite unusual. Twenty seconds or thirty seconds, which would correspond with a distance of 7 km or 10 km, is a more normal figure for the limit of audibility.

In commenting upon the statement of a correspondent at Tung Song, S. Siam, that he had recently, without question, timed thunder to reach him 200 seconds after flashes from a distant storm, and that it was not rare for thunder to be heard 180 seconds after lightning during the distant February and March storms, the editor of the *Meteorological Magazine* quotes 75 seconds as a limit rarely exceeded but cites observations of 255 and 310 seconds on 5 September 1899 at Nordeney noted by Veneema in *Das Wetter*, and refers to a later observation of 600 seconds.

(*Meteorological Magazine*, June 1928, p. 113.)

Considering the vast amount of energy (10^{10} joules) which is released in a flash of lightning, the distance at which thunder is normally audible is surprisingly small. The sound of gunfire or of exploding meteors travels far greater distances.

The curious reverberation of thunder lasting for 15 seconds or even 30 seconds needs some explanation. Its irregularity may be due partly to

the great length of the discharge and partly to the branching of the flashes, to which we shall refer in chap. IX. The discharge may be distributed over several places some miles apart. In a thunderstorm which occurred at St David's flashes several miles in length crossed the zenith.

Transmission in a counterlapse

The special case of an inversion or counterlapse of temperature requires consideration because the temperature may increase considerably with height and the consequent refraction has an important influence. The wave-front in an inversion will be elongated vertically and flattened on the sides, and the energy will be brought more nearly to horizontal transmission, strengthened in the sides, weakened at the top, in a manner which is indicated by the wave-surfaces and rays of fig. 21. Inversions at the surface are common accompaniments of frosty nights and distant sounds are proverbially audible on such nights.

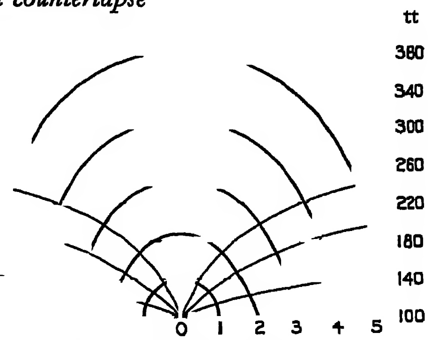


Fig 21 Wave-fronts and rays of sound from a source, O, at the surface, in a counterlapse of temperature, much exaggerated.

LIMITS OF AUDIBILITY. ZONES OF SILENCE

The principles of transmission which have been described have found expression of late years in the investigation of the audibility of the sound of distant guns or other explosions. The subject is treated in a paper by Prof. E. van Everdingen[1] before the Amsterdam Academy in 1916. The general features of the different cases are first a zone of normal audibility within which sound travels along the surface with continuous loss of the local intensity of the energy as represented in fig. 15 until it is so weakened as to be inaudible. Surrounding that is a zone of silence within which the sound is inaudible; and surrounding that again a second zone of audibility which may be called abnormal, and beyond that again may be other zones. The order of magnitude of these zones will be gathered from the information which is cited in the several examples; but making a hasty generalisation the zone of normal audibility is in every way irregular; the inner boundary of the first zone of abnormal audibility is about 160 km from the source. This generalisation of the results of observation can only be regarded as surprising if we consider the matter from the point of view of the energy employed in producing sound. In a lecture before the Royal Institution in 1897 Lord Rayleigh gave 2700 kilometres as the limit of audibility of a fog-signal taking 60 horse-power

[1] 'The propagation of sound in the atmosphere,' *K. Akad van Wetenschappen te Amsterdam*, Proceedings No 6, vol XVIII, pp 933–960, 1915

(45 kilowatts) assuming that the energy is distributed according to the law of inverse square of the distance from the source.

Van Everdingen gives a number of examples obtained on casual occasions in Europe and specially of the sound of volcanic eruptions in Japan. After the war the subject was taken up by the International Commission for the Exploration of the Upper Air and subsequently a special Commission was appointed by the International Meteorological Committee[1]. Investigations have been made in the cases of accidental or deliberate destruction of explosives: at Oppau on 21 September 1921, at Oldebroek in October 1922, la Courtine in May 1924, and at Jüterbog on several occasions between the years 1923 and 1926. The most complete explorations of cases of that kind are described by Ch. Maurain[2] in the discussion of observations of four explosions at la Courtine, and by H. Hergesell in an account of the work of the Commission for the investigation of the sound of explosions which was published in Lindenberg[3] and summarised in the *Meteorologische Zeitschrift* for August 1927.

Explosions at la Courtine

The explosions at la Courtine discussed by Ch. Maurain were on 15, 23, 25 and 26 May 1924, and the zones of audibility on the first three occasions are represented in fig. 22.

The investigation included an inquiry into the nature of the waves recorded upon registering microphones (T.M.) and of the sounds heard.

On peut dire que les tracés donnent l'impression que les ondes enregistrées sont généralement très complexes. C'est d'ailleurs l'impression qu'on retire aussi des comptes rendus des observations à l'oreille; un grand nombre de ces comptes rendus indiquent le son comme double ou multiple, ou comme constituant un roulement plus ou moins prolongé. Presque tous les comptes rendus donnent le son comme sourd, bas, grave. Les ondes paraissent présenter des périodes variant depuis celles de sons graves jusqu'à environ une seconde. Les dentelures signalées dans les exemples ci-dessus correspondent à des périodes de l'ordre de celles des sons graves.—M. Cathiard m'a d'ailleurs indiqué que lorsque les appareils T.M. donnent une élongation brusque, l'onde correspondante à un caractère sonore, même quand le graphique ne présente pas de dentelures à courte période.

On peut se demander si l'onde est complexe dès le début, ou si elle se transforme en se propageant et quel est le genre de cette transformation. Il semble bien que l'onde soit complexe dès le voisinage de son origine.

. . . .

Dans les zones de réception lointaine à caractère anormal indiquées ci-dessus, les ondes ont conservé un caractère sonore accentué. Cela peut s'expliquer d'après le mode de propagation très probable de ces ondes, qui sera discuté plus loin: elles paraissent s'être écartées rapidement du sol, s'être propagées à grande hauteur, et être revenues ensuite au sol; elles n'ont donc pas subi le genre d'amortissement dû aux accidents du relief, et leurs parties à courtes périodes peuvent avoir conservé plus d'importance relative que dans la propagation près du sol.

[1] *Report of the International Meteorological Conference of Directors at Utrecht*, 1923, Appendix H, pp. 160–1, Utrecht, 1924.

[2] *Annales de l'Institut de Physique du Globe*, Paris, 1926, Fascicule spécial. The memoir contains a bibliography of papers by many writers.

[3] *Die Arbeiten der Kommission zur Erforschung der Schallausbreitung in der Atmosphäre*, Lindenberg, 1927.

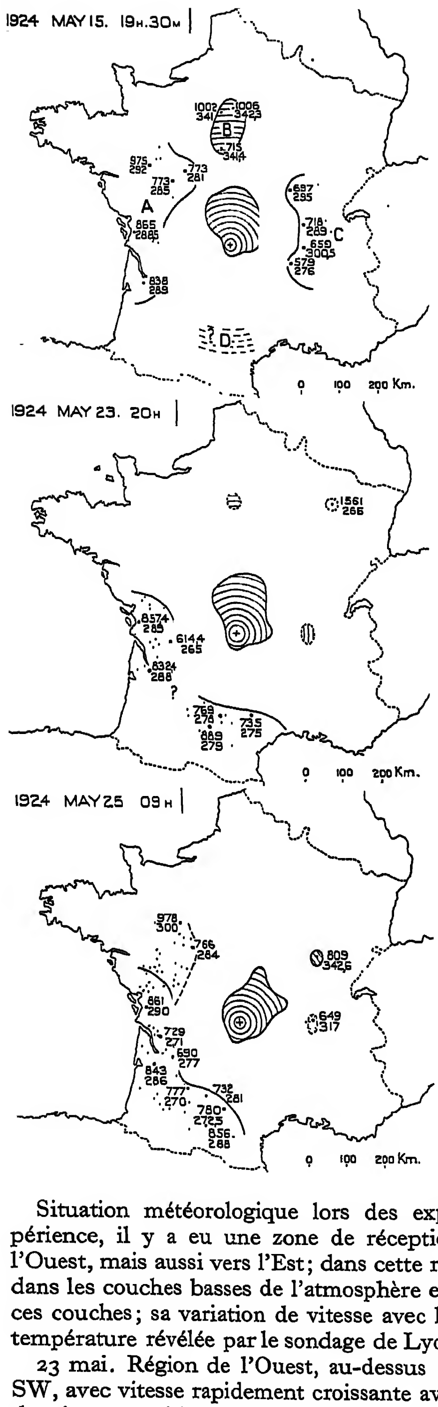


Fig. 22. Zones of silence and zones of audibility on the occasion of the explosions at la Courtine, 15, 23 and 25 May 1924.

The origin of the explosion is marked by a cross +, zones of audibility are marked by circles centred at the + or by stipple. The regions marked by circles are those of "normal reception," i.e. those in which the distance from the origin divided by the time-interval is approximately equal to the normal velocity of sound; the regions marked by stipple are those which received the sound with a velocity sensibly less than the normal.

The figures indicate first the distance in kilometres of the point of reception from the origin of the explosion, and secondly the apparent velocity of propagation of the sound calculated from the distance measured along the surface.

(From Ch. Maurain, 'Sur la propagation des ondes aériennes.')

The results are summarised by Maurain as follows:

Exposé des résultats. Il a été observé, dans chaque expérience, une zone de réception centrale directe, dans laquelle la vitesse de propagation évaluée par rapport à la distance comptée sur le sol est voisine de la vitesse normale du son; cette zone a un développement très différent dans les différentes directions, et, en gros, elle est beaucoup plus étendue dans la direction vers laquelle souffle le vent que dans la direction opposée. Au delà de cette zone, il y a de toutes parts une zone sans réception, puis, dans certaines directions, de nouvelles zones de réception pour lesquelles la vitesse apparente est notablement inférieure à la vitesse normale du son.

Situation météorologique lors des explosions: 15 mai. Dans la première expérience, il y a eu une zone de réception anormale lointaine non seulement vers l'Ouest, mais aussi vers l'Est; dans cette région, le vent soufflait à ce moment du Sud dans les couches basses de l'atmosphère et de l'Ouest ou du Sud Ouest au-dessus de ces couches; sa variation de vitesse avec la hauteur était peu rapide...l'inversion de température révélée par le sondage de Lyon s'étend de 12000 à 14800 mètres environ.

23 mai. Région de l'Ouest, au-dessus de 1000 mètres, le vent était à peu près de SW, avec vitesse rapidement croissante avec l'altitude. Région du Nord, vent de SW de vitesse rapidement croissante avec l'altitude. Région de l'Est (Lyon), au sol

1 mètre N; à 200 mètres 5 mètres N; au-dessus le vent tourne vers E, puis S en croissant constamment; on a ensuite 8 mètres S à 1200 mètres, 9 SW à 1600, 19 SWW à 4000. T = 18°. Région du Midi (Toulouse-Francazal), au sol, calme. Le vent est d'abord NWN et faible 1 mètre, puis il tourne en étant en moyenne d'Ouest, sa vitesse croissant jusqu'à 4000 metres. T = 21°.

25 mai. Région de l'Ouest (Angoulême), au sol 5 mètres SW; le vent reste à peu près SW jusqu'à 2400 mètres en passant par les valeurs, 7, 9, 10, 12, 9, 8, 9. T = 14°. Région du Nord (Le Bourget), vent SSW au sol, SW au-dessus jusqu'à 1600 mètres, croissant d'environ 5 mètres à 13 mètres. T = 13°. Région de l'Est (Lyon), vent NNW 1 mètre; les nuages viennent du S. T = 10°. Temps très nuageux. L'observatoire de Fourvière à Lyon indique au sol un vent d'W modéré, et pour les nuages un mouvement venant de l'W. En d'autres points de la région les nuages sont indiqués comme venant du SW ou de WSW. Région du Sud (Toulouse-Francazal), le vent est faible et à peu près du N jusqu'à 1000 mètres; au-dessus il tourne jusqu'à W en augmentant de vitesse jusqu'à 15 mètres à 2000 mètres. T = 16°.

Explosions at Jüterbog

Two striking examples of normal and abnormal audibility, 3 May 1923 and 26 June 1926, are represented in the illustrations of Hergesell's paper. In that of 3 May 1923 there is a central zone of audibility quite unsymmetrical

RESULTS OF OBSERVATIONS OF THE SOUND OF AN EXPLOSION

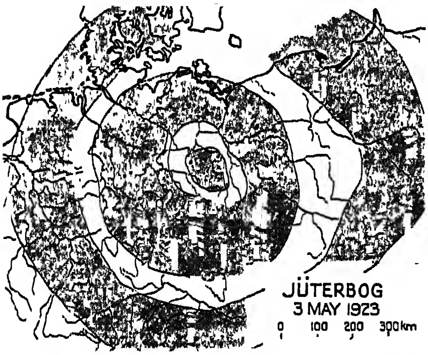


Fig. 23. Charge of 1000 kg at Jüterbog on 3 May 1923. The shaded areas show the regions of audibility.

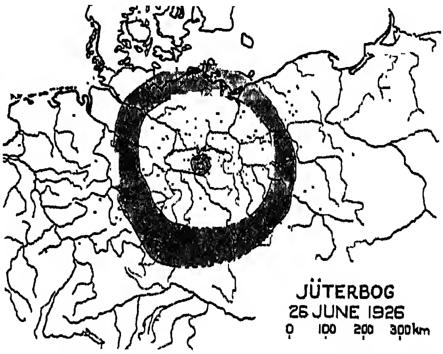


Fig. 24. Charge not stated at Jüterbog on 26 June 1926.

with reference to the locus of explosion. This is surrounded by a zone of silence, that again by a zone of abnormal audibility, another zone of silence and another zone of audibility. That of 26 June 1926 has a very small central zone of audibility symmetrical about the centre, that again surrounded by a zone of silence in this case remarkably broad, and that again by a narrow zone of abnormal audibility.

These shapes are more regular than those which are represented in van Everdingen's paper or in those of other investigators. We must conclude that the distribution of the audibility, though expressing some general principles, is still dependent upon the conditions of the atmosphere on the occasion in ways which are not yet exactly traceable.

Theories of abnormal audibility

It is generally accepted that the central zone of normal audibility expresses the direct transmission of the pulse of pressure along the ground or in the atmosphere lying immediately upon it, that the zone is limited by attenuation of the energy of the sound below the capacity of the human ear or the mechanical substitute for that organ employed by experimenters.

It is also generally accepted that the abnormal audibility beyond the zone of silence is produced by a sound-wave coming from above, diverted from its original upward direction by some feature of the atmospheric structure. We have seen that the refraction of sound is dependent upon the change of velocity of transmission, and that again may be affected by change in the wind-velocity with height, change in temperature, or change of chemical composition.

All these have been invoked to explain the abnormal audibility of sound. Fujiwhara and de Quervain have based the effect on changes in wind-velocity, von dem Borne has cited the supposed transition from the ordinary mixture of gases to helium, hydrogen or geocoronium at such great heights as 100 kilometres (see vol. II, fig. 14). Everdingen came to the conclusion that neither of these would give a quantitative explanation, and indeed the changes of velocity of wind compared with the velocity of sound form rather a shifty foundation for them, and still more shifty is the hypothesis of the absence of mechanical mixture of the atmosphere at great heights which belongs, if it exists, to the unexplored hypothetical. There remains the hypothesis of changes of temperature. That is attributed to Wiechert by Hergesell; it has recently been substantially encouraged by the investigations of Lindemann and Dobson[1] on the phenomena of meteors which ended in the suggestion of the occurrence at 100 km of high temperature, as high or higher than those at the earth's surface. The probability of the existence of such a layer is enhanced by the requirements of wireless telegraphy (II, p. 35). The explanation of the abnormal audibility of sound on this hypothesis has been set out by F. J. W. Whipple[2]. It has been shown already that, if thick enough, a layer of air in which temperature increases with height, must ultimately turn a ray of sound downwards. The thickness required is not great for rays which reach the beginning of an inversion at an angle of incidence of 45° or more. Consequently we may look upon the observations of abnormal audibility as an avenue to the knowledge of the structure of the regions of the atmosphere beyond the range of ordinary meteorology.

The behaviour of a wave-front in an atmosphere made up of two layers of different temperatures (sufficiently different to give a change of velocity of 10 per cent.) without an intervening transition layer is represented in fig. 25.

[1] F. A. Lindemann and G. M. B. Dobson, *Proc. Roy. Soc.* A, vol. CII, 1923, p. 411.

[2] F. J. W. Whipple, *Nature*, vol. CXI, 1923, p. 187, vol. CXII, p. 759; F. A. Lindemann and G. M. B. Dobson, *ibid.* vol. CXI, p. 256; and F. J. W. Whipple, *Meteor. Mag.*, London, vol. LIX, 1924, pp. 49–52; *Second report on Solar etc. Relationships*, International Research Council, Paris, 1929.

O is the source of sound from which spherical waves spread out. Allowing a distance of five millimetres on the diagram between the waves, they reach the surface of change of temperature successively in A_4, A_5,... A_{10}.

From these points secondary waves are set up in the warmer medium in the form of spheres which succeed one another at intervals of 5·5 mm corresponding with the velocity of propagation in that medium. The circles of radius 5·5 mm, 11 mm, 16·5 mm, etc. represent the spherical surfaces reached by the waves. For example, by the time the incident wave has reached A_7 the disturbance from A_4 will have traversed a radius of 16·5 mm, that from A_5 11 mm and from A_6 5·5 mm. The wave-front in the new medium will be the surface which, passing through A_7, touches the sphere of single radius from A_6, of double radius from A_5 and of treble radius from A_4.

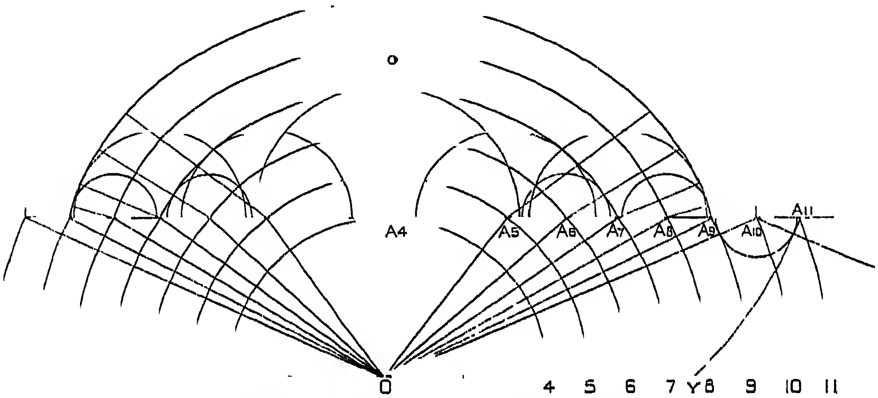


Fig. 25. The return of a sound-wave from an upper layer.

Since the rate of advance in the new medium is greater than in the old and the distances A_4A_5, A_5A_6, are successively smaller and approach 5 mm as a limit, while the radii of the spheres in the new medium are multiples of 5·5 mm, the tangent curve which defines the new wave-front will become steeper as the wave advances in that medium. The energy will be concentrated near the surface of discontinuity as compared with the zenith until a point is reached where the tangent surface is vertical. The figure shows that at A_{10} a vertical plane will touch all three surfaces.

Beyond that point no tangent surface to the wave in this medium can be drawn from the point A_{11} and no further progress on those lines is possible. We must now therefore turn our attention to the disturbance in the original medium as the outlet for the energy of the wave, and assuming that the energy can be disposed in this way we obtain a wave-front of reflexion $A_{11}Y$, and a ray which is directed downwards.

Thus for the transition between the wave-front A_4 and the wave-front $A_{11}Y$ we may invoke the aid of a change of temperature with height so rapid that it is equivalent to discontinuity; its effect is known as total reflexion.

In the lower layers of the atmosphere where the temperature diminishes

with height the wave-front moving from left to right will be turned counter-clockwise and the ray bent upwards as in fig. 19, and only that part of the front which possesses a sufficiently large angle of incidence on the layer of increasing temperature can be turned downwards.

By assuming a distribution of temperature with height it is possible to calculate the path of a ray which leaves the ground sufficiently near to the horizontal to get turned downward at the layer of higher temperature.

F. J. W. Whipple gives a path in a diagram reproduced in fig. 26. He takes a cycloid as the curve in the troposphere, a straight line in the stratosphere and an inverted cycloid in the transition to the high temperature layer.

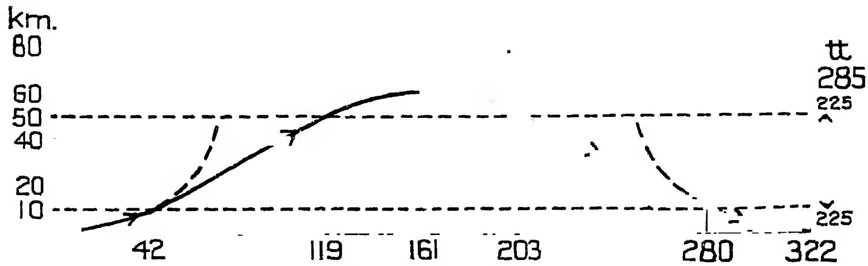


Fig. 26. Path of a sound-ray, originally horizontal, through the troposphere, stratosphere and lower empyrean (*Meteorological Magazine*, vol. LIX, 1924, p. 51).

In the troposphere a uniform lapse-rate of temperature of 6tt per km, from 285tt at the surface to 225tt at 10 km, is assumed, and the path in that region is taken as a cycloid; in the stratosphere with a uniform temperature of 225tt the path is a straight line; in the upper layers with a uniform increase of temperature of 6tt per km from 225tt at 50 km to 285tt at 60 km the path is taken as an inverted cycloid.

In the conditions specified the total horizontal range is 322 km, the time taken by the sound is 19 minutes and the apparent speed 290 m/sec; if the sound could have passed along the earth's surface with the uniform speed appropriate to the temperature 285tt it would have travelled the same distance in 16 minutes.

But if it be indeed atmospheric temperature that accounts for the main features of these interesting phenomena, the lower atmosphere will have a good deal to say about the details of distribution.

A ray of sound passing through a layer of continuously diminishing temperature is bent upwards and will be bent upwards differently on different occasions. But the reversal of the upward direction depends upon the angle of incidence as well as upon the distribution of temperature. Hence we have the possibility of all sorts of effects that are comparable with the effects of lenses upon light.

With the extension of the use of sound for measuring distances and for supplying evidence of atmospheric structure these inquiries form an important part of modern meteorology.

THE IRREGULAR TRANSMISSION OF SOUND

In all the representations of the state of the atmosphere which have been put before the reader in explanation of the features of the audibility of sound we have had in mind a stratification of the atmosphere in layers of continually diminishing or continually increasing temperature. It is hardly necessary to

say that the atmosphere does not allow its behaviour to be completely classified in that way. There are often local patches of inversion of lapse-rate, and nearly always changes of temperature within horizontal layers which have an influence upon transmitted sound comparable with those of the same kind of distribution on the greater scale. All the complications of the structure of the atmosphere come into consideration when the details of transmission of sound are being discussed. It is in the use of sound-transmission for what is called sound-ranging, that is to say the determination of the position from which a distant sound has emanated, that the details of the thermal structure of the atmosphere are of importance. It is to these details that the vagaries of the audibility of fog-horns or other sounds at sea must be attributed.

Now Prof. Tyndall found that from the cliffs at the South Foreland, 235 feet high, the minimum range of sound was a little more than 2 miles, and that this occurred on a quiet July day with hot sunshine. The ordinary range seemed to be from 3 to 5 miles when the weather was dull, although sometimes, particularly in the evening, the sounds were heard as far as 15 miles. This was, however, only under very exceptional circumstances. Prof. Tyndall also found that the interposition of a cloud was followed by an almost immediate extension of the range of the sound. I extract the following passages from Prof. Tyndall's Report:—

"On June 2 the maximum range, at first only 3 miles, afterwards ran up to about 6 miles.

"Optically, June 3 was not at all a promising day; the clouds were dark and threatening, and the air filled with a faint haze; nevertheless the horns were fairly audible at 9 miles. An exceedingly heavy rain-shower approached us at a galloping speed. The sound was not sensibly impaired during the continuance of the rain.

"July 3 was a lovely morning: the sky was of a stainless blue, the air calm, and the sea smooth. I thought we should be able to hear a long way off. We steamed beyond the pier and listened. The steam-clouds were there, showing the whistles to be active; the smoke-puffs were there, attesting the activity of the guns. Nothing was heard. We went nearer; but at two miles horns and whistles and guns were equally inaudible. This, however, being near the limit of the sound-shadow, I thought that might have something to do with the effect, so we steamed right in front of the station, and halted at 3¾ miles from it. Not a ripple nor a breath of air disturbed the stillness on board, but we heard nothing. There were the steam-puffs from the whistles, and we knew that between every two puffs the horn-sounds were embraced, but we heard nothing. We signalled for the guns; there were the smoke-puffs apparently close at hand, but not the slightest sound. It was mere dumb-show on the Foreland. We steamed in to 3 miles, halted, and listened with all attention. Neither the horns nor the whistles sent us the slightest hint of a sound. The guns were again signalled for; five of them were fired, some elevated, some fired point-blank at us. Not one of them was heard. We steamed in to two miles, and had the guns fired: the howitzer and mortar with 3-lb charges yielded the faintest thud, and the 18-pounder was quite unheard.

"In the presence of these facts I stood amazed and confounded; for it had been assumed and affirmed by distinguished men who had given special attention to this subject, that a clear, calm atmosphere was the best vehicle of sound: optical clearness and acoustic clearness were supposed to go hand in hand."

(Osborne Reynolds, 'On the refraction of sound by the atmosphere,' *Proc. Roy. Soc.* No. 155, 1874.)

Osborne Reynolds adds the following comment:

Here we see that the very conditions which actually diminished the range of the sound were precisely those which would cause the greatest lifting of the waves. And

it may be noticed that these facts were observed and recorded by Prof. Tyndall with his mind altogether unbiassed with any thought of establishing this hypothesis. He was looking for an explanation in quite another direction. Had it not been so he would probably have ascended the mast, and thus found whether or not the sound was all the time passing over his head. On the worst day an ascent of 30 feet should have extended the range nearly $\frac{1}{4}$ mile.

For these we have only the changes in the velocity of sound to consider which depend upon temperature and humidity. Temperature cannot alter the velocity by more than 5 per cent., and humidity by not so much as $2\frac{1}{2}$ per cent. With such small differences, however, variations in direction can be explained which are sufficient to account for some of the vexatious experiences of inaudibility. With sound, regarding the zones of abnormal audibility as the equivalent of mirage, we may find analogies to all the phenomena of reflexion, refraction and mirage which are described in the next chapter as exhibited by waves of light.

At the close of chap. v we shall refer to the variations in the local stratification of the atmosphere produced by solar radiation which in this chapter are regarded as the proximate cause of the irregularities of atmospheric acoustics.

SOUNDS OF METEOROLOGICAL ORIGIN

Quite apart from any immediate consideration of the rate of travel or the point of origin there are many phenomena of sound which are definitely associated with wind or weather and ought not to pass unnoticed by the scrupulous meteorologist. In the second edition of his book on the *Physics of the Air*, W. J. Humphreys has devoted a chapter to these phenomena. Therein are included besides thunder, the brontides (mistpoeffers) or "Barisal guns" of the Bay of Bengal which are apparently seismic, the howling of the wind, the humming of wires (with which may be associated the whispering of trees), the murmuring of the forest, the roaring of the mountain and the tornado.

These phenomena have a meteorology of their own better expressed, perhaps, by the poets than by a physical laboratory. In level regions and indeed everywhere except in steep sloping valleys there is stillness in the air before dawn. It is prosaically explained as due to the absence of thermal convexion in a region of counterlapse; but experience teaches us to regard it as natural, and the unnatural sound was too disturbing to be disregarded

> When waken'd by the wind which with full voice
> Swept bellowing thro' the darkness on to dawn.
>
> (TENNYSON, *Gareth and Lynette*.)

CHAPTER III

ATMOSPHERIC OPTICS

C. Huyghens (1629–95), *Traité de la lumière*, 1690.
Thomas Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts*. London, 1807.
A. J. Fresnel (1788–1827), *Œuvres complètes*, 1866–70.
J. D. Forbes, *Supplementary Report on Meteorology. Report of the Tenth Meeting of the British Association for the Advancement of Science*, 1840. London, 1841.
A. Bravais, *Mémoire sur les halos. Journal de l'École royale polytechnique*, XXXIe cahier. Paris, 1847.
R. T. Glazebrook, *Physical Optics*. London, 1883.
E. Mascart, *Traité d'optique*. Gauthier-Villars, 1893.
H. Arctowski and A. Dobrowolski, *Résultats du Voyage du S.Y. "Belgica." Météorologie*. Anvers, 1902, 1903.
J. M. Pernter, *Meteorologische Optik* (completed by F. M. Exner). Braumüller, Wien und Leipzig, 1910 and 1922.
W. J. Humphreys, *Physics of the Air*. Philadelphia, 1920.
F. J. W. Whipple, *Meteorological Optics. Dictionary of Applied Physics*, vol. III. Macmillan and Co., Ltd., 1923.
Sir Arthur Schuster, *An Introduction to the Theory of Optics*. 3rd edition (with J. W. Nicholson), 1924.

We pass on now to deal with waves of light which are the cause of so many impressive phenomena in the atmosphere. At the outset it will be necessary to bear in mind the composite nature of the waves of light. We have mentioned the possibility of complexity in the case of water-waves and sound-waves, but the complexity of the apparent motion of the aether which constitutes light is even more involved. The wave-length of visible light ranges from ·4 micron to ·8 micron[1] and waves of every length between those two limits may be found either in sunlight or in its substitute the electric arc. Sunlight itself is in fact deficient in light of many small groups of wave-lengths, but their absence is attributed to absorption by the sun's external envelope, called its chromosphere, not to the peculiarity of the original source, the photosphere. There are rays, as shown in fig. ii of vol. II, longer than the limits specified; but we deal with those in a subsequent chapter.

If, for the time being, the original nature of sunlight may be omitted from the phenomena which we are called upon to explain, diffused daylight, twilight, the twilight arch, the blue colour of the sky, the colours of the sunrise and sunset should first claim our attention.

The behaviour of light transmitted through a medium of varying density like the atmosphere may be treated in a manner quite similar to that which we have employed for the consideration of the transmission of sound. The changes in the attitude of the wave-front can be related to changes in the velocity of travel of the disturbance. With light the velocity of travel depends on the density of the medium, progress being slower in the denser layers; allowance must also be made for any change in the composition of the medium. For its travel "through space," whatever that may mean, a velocity of 3×10^{10} centimetres per second is assigned, as determined first by the occultations of the satellites of Jupiter, and more or less confirmed by experimental measurements on the earth by Fizeau who used a toothed wheel to determine the time

[1] Micron: $1\,\mu = 10^{-3}$ mm $= 10^{-6}$ m; millimicron: $1\,\mu\mu = 10^{-3}\,\mu = 10^{-9}$ m.
Ångström unit: 1 AU $= 10^{-10}$ m, or tenth-metre.
I should like to plead for symbols for micron and millimicron which would be consistent with the general practice as regards c, g, s, units, and allow the use of μ for refractive index as in the sequel. N.S.

of travel over 6 or 8 miles, and Foucault who used the rotation of a beam by a revolving mirror to indicate the interval for the travel of light over a few metres. The latest determination by Michelson of the velocity of light through space is $2{\cdot}99797 \times 10^{10}$ cm/sec. For meteorological purposes 3×10^{10} cm/sec is near enough.

REFRACTION AND DISPERSION

It is the hypothetical luminiferous aether, free from any ordinary material, which is called upon to carry the energy of light through space at that speed; after it has passed into glass with refractive index μ, it behaves as though its velocity had been reduced in the ratio of 1 to μ. The relation between velocity and refractive index on the hypothesis of transmission by wave-motion without any alteration of frequency of vibration can be established in exactly the same way as that for the transmission of sound on p. 40. Thus for light which passes from one homogeneous medium wherein the velocity is V to another in which the velocity is V', crossing a plane surface where the angle of incidence (i.e. the angle between the ray and the normal to the surface) is i, and the angle of refraction is j, we have the formula $\sin i/\sin j = V/V' = \mu$.

For air at pressure 760 mm (1013mb) and the freezing-point of water, 273tt, $\mu = 1{\cdot}0002918$ for light of wave-length ·5893 micron (Sodium D).

The velocity of this light in standard air is consequently $2{\cdot}99797 \times 10^{10}/1{\cdot}0002918$, i.e. $2{\cdot}997095 \times 10^{10}$ cm/sec. For air of the same composition at other temperatures or pressures $(\mu - 1)/\rho$ is constant, and equal to $\cdot 0002918/\cdot 001293$ or ·2257.

REFRACTIVE INDICES FOR AIR, WATER, ICE AND GLASS

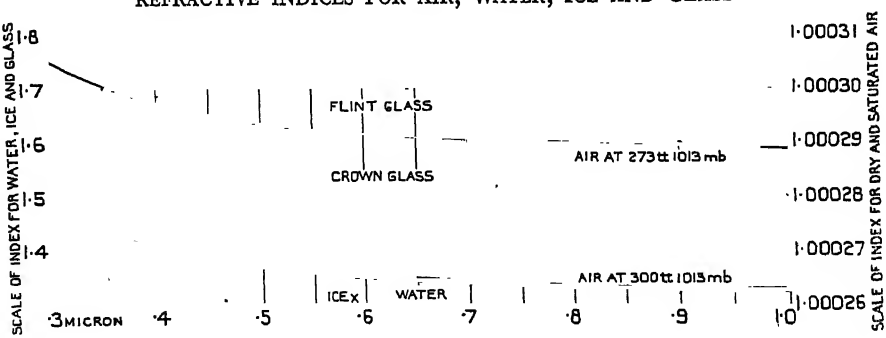


Fig. 27. Curves showing the relation of index of refraction to wave-length according to a scale on the right-hand side for aether to air at two temperatures (the lines are thickened to indicate the difference between dry air and saturated air) and on the left-hand side for air to glass and water and one value for ice.

In every medium except the aether the velocity of light is different for waves of different length. Hence the index of refraction is different and the deviation caused by refraction. The difference for the different colours is called the dispersion of the colours. It is dispersion which produces the spectrum, the order of deviation being infra-red least, red, orange, yellow, green, blue, violet successively greater and ultra-violet greatest.

By the same process, assuming that light is retarded by the atmosphere in proportion to its density, that is to say that its velocity is diminished in proportion to the mass of the atmosphere which a beam of definite area has to cross, we can explain the apparent displacement of the heavenly bodies as by refraction, according to a formula which is set out by Lord Rayleigh

$$\delta\theta = (\mu_s - 1)\left(1 - \frac{H}{O}\right)\tan\theta - (\mu_s - 1)\left(\frac{H}{O} - \frac{\mu_s - 1}{2}\right)\tan^3\theta,$$

where θ is the zenith distance as observed, μ_s the refractive index of air at the surface (depending on its density since $\mu - 1$ is directly proportional to the density), H the height of the homogeneous atmosphere, and O the earth's radius.

"If $H \doteq 7{\cdot}990 \times 10^5$ cm, $O = 6{\cdot}3709 \times 10^8$ cm, and $\mu_s - 1 = {\cdot}0002927$, all closely approximate values, then

$$\delta\theta = 60{\cdot}29'' \tan\theta - 0{\cdot}06688'' \tan^3\theta.$$

This is Lord Rayleigh's final equation, and it appears to be exceedingly accurate for all values of θ up to at least 75°, or as far perhaps as irregular surface-densities generally allow any refraction formula to be used with confidence[1]."

For a star at an elevation of 45° the correction for refraction would be 60·22″.

The calculation of the effect of refraction is much simplified if we disregard the curvature of the earth and of the layers of atmosphere above its surface. The phenomena are then treated as belonging to a series of plane layers of air of which the density diminishes with height. In that case $\delta\theta$ the deviation of the apparent direction from the true direction outside the atmosphere is given by the refractive index at the surface $\sin i = \mu_s \sin j$,

$$\sin i - \sin j = (\mu_s - 1)\sin j,$$

and if all the angles are small, i.e. for objects near the zenith,

$$i - j = (\mu_s - 1)j.$$

Since the effect depends upon $\mu_s - 1$, which is proportional to the density of the surface-layer, it ought not to be regarded as the same in all places or on all occasions.

The effect of refraction is most conspicuous in the visibility of the sun at rising or at setting when the sun itself is below the horizon. The wave-front of the light coming from the sun is bent downwards towards the earth because the travel of the light is faster in the upper air than it is close to the earth where the density of the air is greatest. The swinging forward of the front may be compared with that of an advancing wave which curls round to face a shelving shore as described on p. 26 because the motion of the lowest layer is retarded.

Lord Rayleigh's formula is not applicable beyond 75° and cannot therefore be applied to calculate the amount of refraction of a ray on the horizon and the consequent extension of daylight beyond the astronomical day. In practice an empirical formula, due to Bessel, is used, which for pressure 1013mb and temperature 283tt gives approximately 34′ below the horizon as the angle of incidence of the first and last rays of sunlight, subject to correction for departure from the normal of the density of the surface layer of

[1] W. J. Humphreys, *Physics of the Air*, 1920, p. 439.

air. The range of density at the surface may correspond with 100mb and 100tt altogether, and assuming that the effect is proportional to the change of density the allowance of elevation might vary from 30 minutes to 48 minutes of arc.

In any ordinary case the extension is small, but on occasions it has been noticed. Pernter cites a case in which the sun and moon were both visible during an eclipse of the moon, when according to astronomical calculations sun, moon and earth were in line. Either the sun or the moon must have been below the geographical horizon. The elevation of each of them through 15′ of arc would have brought both above the horizon, and that is well within the limits which we have computed.

In the formula for the length of daylight which we have quoted in vol. I, p. 44, as the result of observation, an increase of the sun's elevation of 34′ is allowed; that is to say the centre of the sun is regarded as visible when the ray coming from it is at 34′ below the horizon. From that formula the duration of insolation has been computed in the International Tables. We give the following table to show the effect of refraction upon the duration of visibility of the sun in different latitudes for different times of the year according to the sun's declination.

It will be noticed that at the equator the sun's day is ·08 hour longer than the twelve hours which should correspond with the geometry; that allows 2½ minutes sunlight, morning and evening, on account of refraction. In latitude 80°, on account of the small inclination to the horizon of the sun's apparent path in the sky, the allowance at the equinoxes is thirteen minutes, ·21 hour, morning and evening.

Duration of insolation in different latitudes for different values of the sun's declination δ

Latitude 0°	10°	20°	30°	40°	50°	60°	70°	80° N	δ
h	h	h	h	h	h	h	h	h	
12·00	12·58	13·21	13·93	14·85	16·15	18·50	24·00	24·00	23° 27′ N
+ ·08	·08	·09	·10	·12	·15	·25	—	—	
12·00	12·49	13·02	13·62	14·37	15·43	17·21	24·00	24·00	20° 00′ ,,
+ ·08	·08	·08	·10	·11	·14	·21	1·90	—	
12·00	12·38	12·80	13·27	13·86	14·67	15·97	18·94	24·00	16° 00′ ,,
+ ·08	·08	·08	·09	·11	·13	·18	·38	—	
12·00	12·28	12·59	12·94	13·37	13·96	14·88	16·77	24·00	12° 00′ ,,
+ ·08	·08	·08	·09	·10	·12	·17	·28	—	
12·00	12·19	12·39	12·62	12·90	13·28	13·87	15·02	19·10	8° 00′ ,,
+ ·08	·08	·08	·09	·10	·12	·16	·24	·73	
12·00	12·09	12·20	12·31	12·45	12·63	12·93	13·47	15·13	4° 00′ ,,
+ ·08	·08	·08	·09	·10	·12	·15	·23	·48	
12·00	12·00	12·00	12·00	12·00	12·00	12·00	12·00	12·00	0° 00′ ,,
+ ·08	·08	·08	·08	·10	·12	·15	·22	·42	

The table is made out for the northern hemisphere and for the sun north of the equator. The first line gives the number of hours according to geometrical calculation, the second line the fractions of an hour added by refraction. To obtain the duration of insolation for the same hemisphere when the sun is south of the equator the values given in the upper lines of the table must be subtracted from 24·00; the allowance for refraction is unaltered.

The table applies equally for the southern hemisphere for the sun south of the equator.

For example to find the duration of insolation in London on 30 Sept. 1924, we have latitude 51° 28′ N, declination δ 2·8° S, whence interpolating for the two columns we get 24·00 − 12·47 + ·12 = 11·65 h.

The geometrical horizon

In meteorological practice the refraction of light by the atmosphere near the surface shows itself in various interesting ways which differ according to the variations of temperature, and consequently of density in the surface layers. The results of the refraction are exhibited as the "looming" of distant objects, the lifting or lowering of the visible horizon, and the various forms of mirage. The phenomena are frequently seen at sea and the distant objects which are apparently displaced or distorted are ships, islands or icebergs with the horizon to which they belong. One of the common effects is that refraction brings into view an object which is actually below the true horizon at the time. Thus the visible horizon when there is great variation of density near the surface may be a very deceptive object. It may therefore be convenient to give here a diagram (fig. 28) from the *Meteorological Glossary* which gives the distance of the visible horizon from a point of specified

THE DISTANCE OF THE GEOMETRICAL HORIZON FOR DIFFERENT HEIGHTS

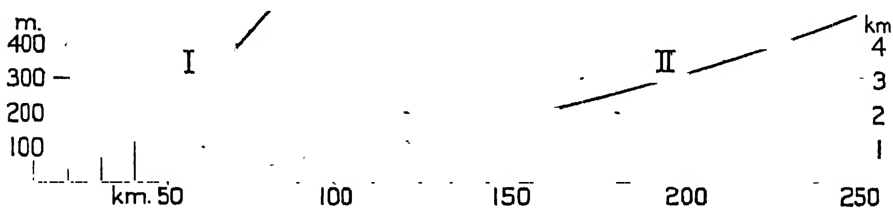


Fig. 28. Showing the relation between the height in metres of a point of observation and the distance of the horizon, making no allowance for refraction; or between the height in metres of a cloud or other distant object and the distance in kilometres at which it is visible on the horizon from a point at sea-level. Curve I refers to the height scale on the left, Curve II to that on the right.

elevation when no allowance is made for refraction, or, what is practically the same figure, the distance from which an object of specified height is visible at the surface. In order to bring the diagram within suitable limits two curves are given relative to two scales of height, but the results as read from either curve are the same.

Looming and superior mirage

Distortion and apparent dislocation of objects near the surface can be produced by the coldness of the surface in relation to the layers of air above it. The travel of light in the surface layers is in consequence retarded.

By the retardation the wave-front of a beam of light from a point just above the earth's surface is bent forward, and consequently an observer at Q looking at a distant object P will see it with an apparent elevation greater than the actual, exceptionally greater if the density of the surface layer is exceptionally great.

In that case the atmosphere will act as a prism as indicated in fig. 29. In the case of the prism a plane wave-front travelling along the horizon and

falling on the face which is nearest the object will emerge as a plane wave-front sloped forward. The ray in consequence will be bent downwards and appear to the observer to come from a point above the corresponding point of the object. The difference between the atmospheric effect and the prismatic analogy is that in the atmosphere the turning of the front is due to the velocity of light being less in the layers of greater density near the ground. In the prism the velocity of light is less in the glass than in the air but the same in

Fig. 29. The deviation from the vertical, much exaggerated, of a plane wave-front in an atmosphere in which density decreases with height. A similar effect produced by a prism of small angle is shown for comparison.

any part of the glass; the turning of the front is due to the different lengths of path through the glass of different parts of the front. Each path causes a lag, and there is greater lag for the longer path near the base. The end of either process is a greater lag in the lower part of the wave-front which was originally vertical. The prism will indeed produce the same lag as a certain length of atmosphere with its lapse of density in the vertical, provided of course that the lapse of density in the air corresponds with the lapse of distance traversed in the prism. In fig. 29 the prism is drawn with plane faces; that is only justified for an atmosphere with uniform lapse of velocity of travel. Changes of density in the atmosphere are naturally irregular and half a cylindrical lens is a more apt substitute than a prism with plane sides.

With half a cylindrical lens distant objects will appear nearer to the observer, exaggerated in size and elevation. By the corresponding process objects below the horizon may become visible. Conditions are favourable for the appearance when warm air lies on ice-cold water, and the effect is well known to sailors as "looming."

This is a form of mirage which has been distinguished as superior mirage, and is characteristic of notable counterlapse of temperature, sharp inversion of temperature-gradient, at the surface.

A classical example is described in the following extract:

July 26, about 5 o'clock in the afternoon, while sitting in my dining-room at this place, Hastings, which is situated on the Parade, close to the sea-shore, nearly fronting the south, my attention was excited by a great number of people running down to the sea side. On inquiring the reason, I was informed that the coast of France was plainly to be distinguished with the naked eye. I immediately went down to the shore, and was surprized to find that, even without the assistance of a telescope, I could very plainly see the cliffs on the opposite coast; which, at the nearest part, are between 40 and 50 miles distant, and are not to be discerned, from that low situation, by the aid of the best glasses. They appeared to be only a few miles off, and seemed to extend for some leagues along the coast. I pursued my walk along the shore to the eastward, close to the water's edge, conversing with the sailors and fishermen on the subject. At first they could not be persuaded of the reality of the appearance; but they soon became so thoroughly convinced, by the cliffs gradually appearing more elevated, and approaching nearer, as it were, that they pointed out, and named to me, the different

places they had been accustomed to visit; such as, the Bay, the Old Head or Man, the Windmill, &c. at Boulogne; St. Vallery, and other places on the coast of Picardy; which they afterwards confirmed, when they viewed them through their telescopes. Their observations were, that the places appeared as near as if they were sailing, at a small distance, into the harbours.

Having indulged my curiosity on the shore for near an hour, during which the cliffs appeared to be at some times more bright and near, at others more faint and at a greater distance, but never out of sight, I went to the eastern cliff or hill, which is of a very considerable height, when a most beautiful scene presented itself to my view; for I could at once see Dengeness, Dover cliffs, and the French coast, all along from Calais, Boulogne, &c. to St. Vallery; and, as some of the fishermen affirmed, as far to the westward even as Dieppe. By the telescope, the French fishing-boats were plainly to be seen at anchor; and the different colours of the land on the heights, with the buildings, were perfectly discernible. This curious phenomenon continued in the highest splendour till past 8 o'clock, though a black cloud totally obscured the face of the sun for some time, when it gradually vanished. I was assured, from every inquiry I could make, that so remarkable an instance of atmospherical refraction had never been witnessed by the oldest inhabitant of Hastings, nor by any of the numerous visitors come to the great annual fair. The day was extremely hot. I had no barometer with me, but suppose the mercury must have been high, as that and the 3 preceding days were remarkably fine and clear. To the best of my recollection, it was high water at Hastings about 2 o'clock p.m. Not a breath of wind was stirring the whole of the day; but the small pennons at the mast-heads of the fishing boats in the harbour were in the morning at all points of the compass. I was, a few days afterwards, at Winchelsea, and at several places along the coast; where I was informed, the above phenomenon had been equally visible. When I was on the eastern hill, the cape of land called Dengeness, which extends nearly 2 miles into the sea, and is about 16 miles distant from Hastings, in a right line, appeared as if quite close to it; as did the fishing-boats, and other vessels, which were sailing between the 2 places; they were likewise magnified to a great degree.

('On a singular instance of atmospherical refraction.' By Wm. Latham, Esq., F.R.S., and A.S. *Philosophical Transactions*, 1798, p. 357.)

The idea of the phenomenon being due to exceptional counterlapse of temperature resulting from warm air over cold sea is supported by an example quoted by Pernter from Captain Cook's South Polar voyages.

Position by observation ♀ Dec. 24, 1773 Lat. 67 3 Long. E. of Greenwich 223 0½, Therm. 33.
☿ Jan. 5, 1774 Lat. 52 12 Long. E. of Greenwich 224 45, Therm. 46.
Journal of situation ♀ Dec. 31, 1773 Lat. 59 40 Long. E. of Greenwich 225 11.

1773 Dec.	Morn. Therm.	Noon Barom.	Noon Therm.	Even. Therm.	Winds	Weather, &c.
♀ 31	33	29·05	35½	35	Variable	Little wind and cloudy with sleet at times

*** To-day while we were observing the meridian altitude of the sun a shower of snow came from the west, and passed ahead of the ship; during which a large island of ice, considerably within the visible horizon, and directly under the sun, was entirely hid by it; yet the horizon appeared as distinct, and much the same as it usually does in dark hazy weather. When the shower was over, I found that it required the sun to be dipped something more than his whole diameter to bring his lower limb to the nearest edge of the ice-island, which must have been further off than the visible horizon, during the shower; and yet this would have been taken as the real horizon, without any suspicion, if it had been everywhere equally obscured. Hence may be inferred the uncertainty of altitudes taken in foggy, or what seamen in general call, hazy weather.

1774 Jan. 30	Morn. Therm.	Noon Barom.	Noon Therm.	Even. Therm.	Wind	Weather, &c.
⊙	31½	28·8	32	32½	ENE	Mod. wind and foggy with snow

Position 70·45 S, 253·29 E.

*** This morning we discovered a prodigious large field of ice right ahead, extending east and west farther than could be seen from off the main-topgallant yard. At a distance, the whole appeared very high, and like one solid fixed mass, with many exceedingly high, mountainous parts in it; but when we came nearer, we found its edge, which before appeared upright, and of one solid piece, scarce higher than the water, and composed of many small pieces, close joined together, with some pretty large ice-islands amongst. Farther in, it yet appeared high and mountainous; but probably this also was a deception caused by the very great refractive power of the atmosphere, near the horizon in these frigid regions; many instances of which I had occasion to mention in the account of my Voyage to, and residence in, Hudson's Bay. Let me add here, once for all, that I have had abundant proofs of the effects of these extraordinary refractions on altitude of the sun etc taken from the horizon of the sea with Hadley's quadrant this voyage. For, universally, I believe without a single exception, the east longitude shown by the watch K in the morning, fell short of that deduced from it in the afternoon, when both were reduced to the same mean time by the log, and that sometimes by 10, 12 and even 15 minutes of longitude: I mean when we were in high latitudes for, between the tropics, I seldom knew them differ by more than 3 minutes and not often so much as that.

(The thermometer used was thought to register 33° at the freezing-point or 33½.)

(The Original Astronomical Observations made in the course of a voyage towards the South Pole and Round the World in H.M.SS the Resolution and Adventure in the Years MDCCLXXII,.III,.IV,.V, by William Wales F.R.S., Master of the Royal Math^cl School in Christ's Hospital, and Mr. William Bayly late assistant at the Royal Observatory. Published by order of the Board of Longitude at the expense of which the observations were made. London MDCCLXXVII. p. 351.)

Similar phenomena have also been observed on land. See Pernter, *Meteorologische Optik*, 1902, p. 74 et seq.

If the increase of temperature with height is sufficiently rapid and extensive there will come a stage resembling that which we have already described as causing the return to earth of the sound-waves. The wave-front issuing from a distant point will be divided into two parts; one part reaches the observer as a diverging pencil of rays and the other part as a converging pencil. The former part will give the looming of a refracted image and the latter an inverted image.

Such images are described by Vince and Scoresby[1]. The most noteworthy are triple images of a distant ship; of the three one appears on the sea, the other two in the sky, the upper one erect and the lower inverted.

It is to be remarked that the notable counterlapse of temperature at the surface will be appealed to subsequently (vol. IV, chaps. IV, V) as representing a condition suitable for the formation of fog at sea when the warmer air travels over colder water. In fact superior mirage and fog are at least first

[1] *Phil. Trans. Roy. Soc. London*, 1799, p. 13; *Trans. Roy. Soc. Edin.* vol. VI, p. 245, vol. IX, p. 299.

cousins if not twin brother and sister. In the logs which record superior mirage there is generally a simultaneous reference to fog. The formation of fog would of course prevent the appearance of "looming" and the conditions of occurrence of the optical phenomena are to that extent limited.

Artificial mirage

By interposing between an object and a camera a rectangular cell of sugar solution, with its density suitably graded, Arnulph Mallock[1] has obtained photographs which exhibit the triple images, two erect and one inverted, such as those described in the works referred to. The cell is first partly filled with water, then sugar solution is introduced at the bottom and the distribution of density is determined by natural diffusion. We have reproduced the photographs (fig. 30) because the conditions can be defined and by them we can trace the physical process of the triple mirage.

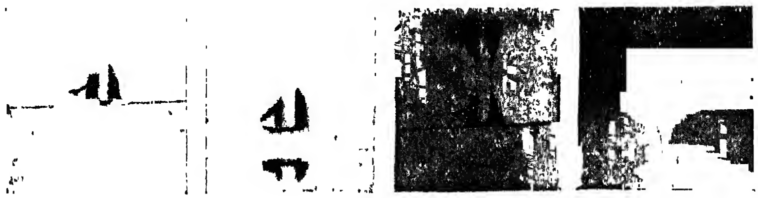


Fig. 30. Photographs of the shape of a ship through a cell of water and syrup from the positions E_4, E_3 and E_1. (Cp. fig. 31 *a*.)

By using another sample of the same solution in a hollow vertical prism and observing the distortion produced by it in the image of a vertical line of light Mallock obtained the variation from layer to layer in the refractive index of the light and hence in its velocity in the solution. The effect of the contents of the cell upon a parallel beam is represented in fig. 31 *a*.

The curve which represents the wave-front is roughly separable into four parts, first a basal portion of maximum lag, due to sugar solution of uniform density, second a portion, concave to the observer, representing rapid transition from strong solution to uniform change, third a portion representing rapid transition from uniform change to plain water, and finally a portion with only the minimum lag of plain water. The original plane wave-front becomes therefore a front which is divisible into four parts corresponding with those of the curve of refraction:—the basal part by which the object is seen in its natural size and position if the camera is within the limits of the basal layer, a concave portion of converging rays which shows an inverted image in a camera placed beyond the crossing of the converging rays, a convex portion of diverging rays which shows a magnified erect image nearer to the observer than the object itself, and the layer of clear water on the top through which some portion of the object (if it is high enough) will be visible to a camera itself at the level of the object. Leaving the last or mast-head vision out of

[1] *Nature*, vol. CXXII, 1928, p. 94.

account we can make use of Mallock's analysis to explain the effect of the distorted wave-front. Placed in a suitable position the camera shows the images formed by the other three parts of the wave-front. The inverted image is ill-defined and there may be vertical elongation or contraction in that as well as in the upper erect image.

Relying upon Mallock's experiment we can trace the formation of the three images. The seeing of the object directly through the layers of uniform density cw, wa at the top and the bottom of the cell needs no explanation. We may draw special attention to the formation of the images by rays which traverse the curved portions ba, bc of the final wave-front ww.

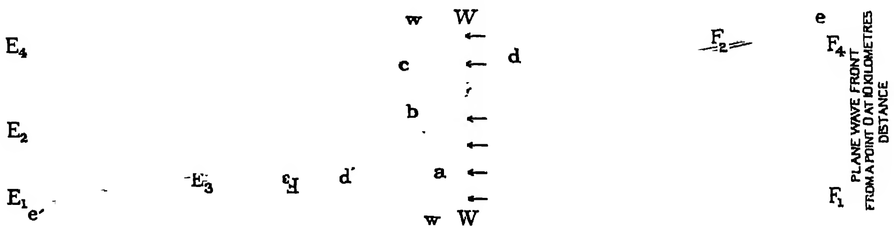


Fig. 31 *a*. The distorted wave-front ww derived from an original plane wave-front WW by the interposition of a cell of sugar solution of graded density, and the formation of images of a distant object by the distorted wave-front.

The incident parallel beam may be taken as coming from an object 3 metres from the cell and the depth of solution in the cell upon which the incident beam falls as 10 cm. WW is the initial wave-front incident upon the cell, ww the wave-front after traversing the solution by which it is divided into cw plane front, cb curved front with rays diverging from successive points of the caustic de, ba curved front with rays convergent to successive points of the caustic e′d′ and aw plane front. An eye at E_1 will see an erect image along the horizontal at F_1, a direct image at F_2 and an inverted image at F_3 between the cell and the eye. An eye placed at E_2 will see only an image near F_2, and one at E_3 will see an erect image at F_1 and an inverted image at F_3, an eye at E_4 will see only an erect image at F_4.

The portion ba concave to the observer which comes from the lower middle of the cell has a varying curvature, and the rays which are normal to that part of the front do not all meet in a point to form a perfect image but are tangential to successive portions of a curve called a caustic[1] represented by d′e′; an image will appear to be at that point of the caustic from which the rays which reach the eye diverge. Similarly the rays normal to the other curved portion behave after leaving the cell as though they came from successive points of the curve de which is also a caustic. What the eye or the camera recognises as an image will therefore appear to be at some point above d′e′ in the one case and some point below de in the other; but in any case the actual distance from the eye is so great that the differences of distance do not specially attract the attention of the camera, but in the atmosphere the displacement towards the observer of the erect image explains the apparent magnification called looming, and the position of the inverted image explains its want of definition.

One other point ought to be noticed, namely, that of the three images simultaneously visible two are erect and one inverted. The explanation which we have given deals only with the formation of the images of a single point. To get the images of the different points of a distant object we must imagine

[1] The formation of a caustic is illustrated in fig. 39.

the picture and the apparatus which it represents rotated through the angle required to direct it towards the selected point of the object. It will be understood that if we direct the whole apparatus, cell and camera, towards a point of the object above the point first selected, we shall turn the images formed through the same angle. Consequently the images on the side of incidence upon the cell will be lifted and that on the side of emergence will be depressed. In the former case the images are erect and in the latter inverted.

In accordance with an explanation already given we could use a glass refractor in place of Mallock's cell if it could be suitably shaped. The shape required to give the wave-front indicated would be that represented in fig. 31 *b*. The upper part is equivalent to a concave lens, the lower part to a convex lens each with inclined axis, the top and bottom are plane. The figure also includes a sketch of the wave-front which the shaping of the glass would produce if exposed to an incident plane wave.

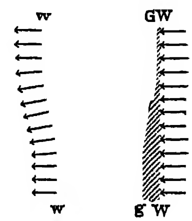


Fig. 31 *b*. Gg a glass substitute for a sugar cell to produce a wave-front ww from a plane front WW.

In applying the explanation to natural phenomena it ought to be remarked that the atmosphere is in no way bound to produce a wave-front which is the exact counterpart of that produced by Mallock's cell. Many forms of wave-front must be allowed; the one which we have used in our drawing, following Mallock, is appropriate to some one distance of the object for a particular distribution of temperature.

MIRAGE IN THE RED SEA. 18 MAY 1928

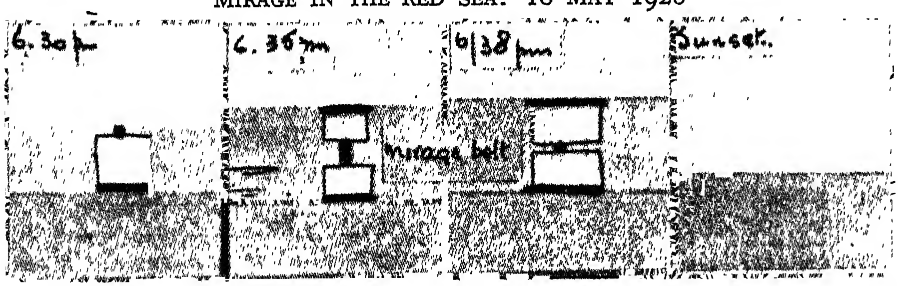


Fig. 32. False horizon above the true horizon in the Red Sea lat. 26° 58′ N, long. 34° 27′ E, with a passenger steamer approaching from the south. From a sketch in *The Marine Observer*, vol. VI, p. 104, reported by R. A. Kneen, 3rd officer, S.S. *Stockwell*: "At sunset 6 h 43 m the horizon appeared normal and the distortion ceased."

The Arctic regions, with their facilities for producing a cold surface, are a natural home of the superior mirage. But in the classical example which we have quoted it was the air over the water of the English Channel that produced the remarkable visibility and looming of the opposite French shore, and examples are to be found in many seas. A number have been represented in *The Marine Observer*—we give a reproduction (fig. 32) of one seen in the Red Sea on 18 May 1928.

Inferior mirage

Phenomena which are the reverse of those described as characteristic of the superior mirage can be seen when the density of air increases with height in consequence of exceptional lapse-rate in the surface-layers. In vol. II, p. 54, we have noted the possibility of lapse-rates much in excess of the adiabatic lapse for dry air, when the surface has been subject to strong sunshine for a considerable time. The figures there mentioned gave a lapse-rate of 25tt for 4 feet, which would correspond with a rate of 20tt per metre, two thousand times the adiabatic limit for dry air and hundreds of times the auto-convective lapse-rate.

In all such cases there is a very rapid increase of density with increase of the distance from the hot surface, and the propagation of the light-waves along the surface is appreciably faster than at a distance from it. The wave-front of a beam of light which is incident obliquely on the heated layer becomes in consequence accelerated in its lower part and bends round so much as to acquire an aspect upwards instead of downwards, and consequent reflexion of the beam. It thus gives rise to an inverted image as if the surface were a reflector:—a mirage—which is called inferior because the fictitious image is below the visible surface. Since the hot surface behaves like a mirror, it reflects the light from the sky above the distant horizon and the objects in the foreground; the surface is in consequence indistinguishable from a water-surface.

The behaviour of the wave-front in this case is illustrated by fig. 33.

The waves which actually diverge from the point O after distortion by the irregularities of the lower layers behave at E as though they diverged from a point O′.

The diagram is constructed to indicate a principle that is generally applicable in problems of this kind, namely, that the wave-front at any time is a surface which connects the positions at which light starting from a point would arrive simultaneously by all the different paths which are possible. Thus the time of travel from O to E is the same for any portion of the front. The travel in the lower path being nearer the ground is through less dense air and the rate is greater than in the upper path which is in cooler air, and the longer distance indicated can be described in the same time as the shorter.

Fig. 33 (*a*) shows the experience of a wave-front in which a pencil diverging from O remains divergent at E and there has its centre at O′. The formation of a real image at O″ on the same principle is shown in fig. 33 (*b*) in which the lower ray gains so rapidly over the upper one that the lowest part of the front gains distinctly more than its neighbours and the front becomes convergent. The image is at O″ and is real. The eye at E will see an erect image in the first case and an inverted one in the second case.

The process can be followed on the same lines as fig. 31, inverted to show a medium in which density increases with height above a hot surface, and *mutatis mutandis*, bearing in mind the inversion, the conclusions are applicable.

But in ordinary circumstances the colder air over a hot surface is in a very unstable condition compared with that of warm air over a cold surface, and consequently the phenomena of inferior mirage are the more likely to be transient and changeable.

INFERIOR MIRAGE

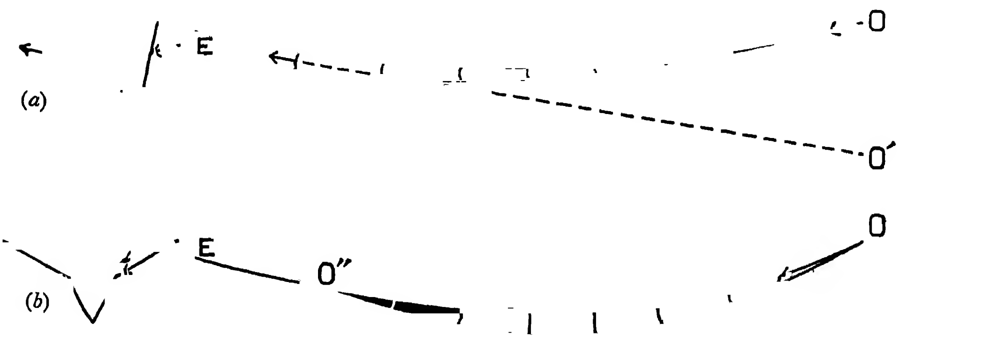


Fig. 33. Pencil of rays and wave-fronts illustrating the formation of virtual and real images.

The inferior mirage is very common in flat desert regions during the warmer hours of the day. It is generally accompanied by "shimmering," a familiar appearance over any approximately regular surface in sunshine. Shimmering is attributed to the irregular variations of density which are associated with the process of convexion of the highly heated air. It confirms the illusion of the mirage as the appearance of water with a slight ripple.

W. J. Humphreys cites an example of a mirage of this kind in Mesopotamia on 11 April 1917, about which General Maude reported officially that fighting had to be temporarily suspended on that account.

In his work on the atmosphere, translated in 1873 by James Glaisher, Camille Flammarion devotes a chapter to descriptions of mirage, mostly the inferior mirage of the African desert, with an illustration of apparent pools in the distance; he has a quotation from Diodorus Siculus in the first century B.C., in which the phenomena of the mirage are described, not very accurately, as an extraordinary phenomenon which occurs in Africa at certain periods especially in calm weather. He recites other impressive examples in Egypt and Algeria.

R. W. Wood[1] has used a surface artificially heated to produce a model mirage in which representations of palm trees appear inverted as by reflexion from a water-surface.

Since the introduction of the practice of making roads with a very smooth surface the inferior mirage is a very common experience on a sunny day. Quite recognisable reflexions are often seen, and people appear to be walking through pools of water[2]. The details of lapse-rate require investigation.

[1] *Physical Optics*, The Macmillan Co., New York, 1911, p. 90.

[2] L. G. Vedy, 'Sand mirages,' *The Meteorological Magazine*, vol. LXIII, 1928, p. 249.

A mirage on the Mall in St James's Park, London. is shown in the photograph of fig. 34.

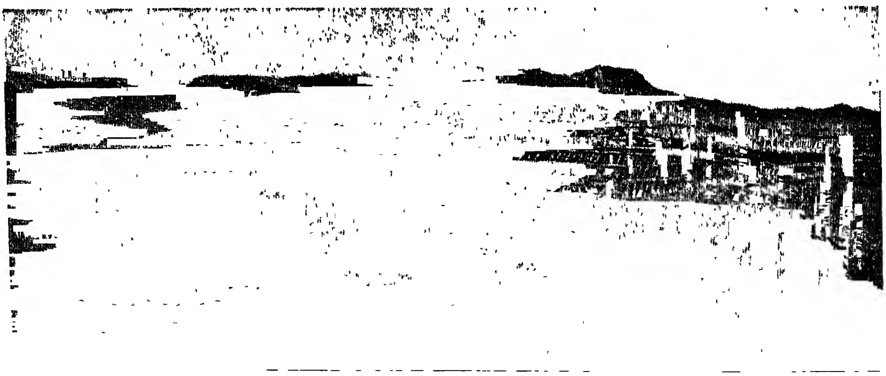
These and other phenomena of mirage can be imitated practically by the effect of glass prisms or plates or combinations of the two, on the understanding that the thickness of the glass to be traversed delays the travel of a wave-front in the same way as the excess of density of air in a layer of the atmosphere retards the part of the wave-front that travels along it, as compared with another part travelling in a layer of less density. In the several illustrations of the inferior mirage, as in those of the superior mirage to which we have given our attention, an approximate optical equivalent in glass is also represented.

Fig. 34. Mirage in the Mall, London, 1921.

Fata Morgana

One form of mirage, which probably combines the characteristics of both inferior and superior, is known in the Mediterranean as the Fata Morgana, or Morgan the Fairy, so named from the half-sister of King Arthur to whom legend has assigned the possession of the palaces suggested by the effects of refraction upon projecting objects of the distant landscape. It is often seen as the looming of one side or other across the Straits of Messina (fig. 35).

Fig. 35. A view of the town of Reggio and the Straits of Messina under the conditions accompanying Fata Morgana. (From Pernter's reproduction of an original by P. Antonio Minasi.)



Prof. F. A. Forel in an address before the Royal Society of Edinburgh in July 1911 described the conditions under which he had observed the Fata Morgana "in the springtime year after year over the Lake Leman. His general conclusions are: (a) The Fata Morgana is made manifest at the region where the morning type of refraction in air over warm water is being transformed into the afternoon type of refraction over cold water. (b) At this region the eye of the observer placed at a convenient height sees simultaneously and in superposition both the depressed and the elevated horizons

associated with the two types of refraction. (c) Bright objects on the lower parts of the opposite coast are stretched and drawn out in height between the two momentarily coexistent false horizons of the lake, and, by forming rectangles in juxtaposition, give the appearance of the banded or ribbed structure of the striated zone. In his memoir of 1896 he showed that the transition from the one type to the other does not take place slowly and progressively: that even when towards the middle of the day the temperature of the air becomes equal to that of the water, and ere long slightly exceeds it, the depression of the apparent horizon and other mirage phenomena associated with the refraction over warm water persist for some little time. During the persistence of this mirage over the cold water there must be an unstable equilibrium due to the thermal stratification in the lower layers of air. The rapid transformation from this instability to the stability associated with the direct thermal gradient is the determining factor in the production of the Fata Morgana. The suddenness of its appearing and its brief transitory character are at once explained."

(*Q.J. Roy. Meteor. Soc.* vol. XXXVIII, 1912, p. 219.)

The relation of temperature to refraction

In the preceding sections we have discussed the apparent displacement of distant objects which is contingent upon the variation of temperature and consequent variation of density in the layers near the surface. So far as the atmosphere is concerned our treatment of the subject has been expressed merely in general terms, but it is evident that there must be a numerical relation between the displacement of the image from the position of the object and the variation of temperature in the layers which produce the observed effect.

We have used the term lapse-rate to indicate the rate of fall of temperature with height and have called an increase of temperature with height a counterlapse. It would appear at first sight that it should be possible to trace a numerical relation between the displacement of a distant object the height of which is known and the lapse or counterlapse of temperature by which it is produced, but in this connexion D. Brunt[1] has noted that the refractive index of water-vapour is 1·000257, i.e. ·000035 less than that for dry air. For a mixture of 1 gramme of air with x grammes of water-vapour the value of the refractive index, μ, is given by the equation

$$(\mu - 1)\, tt p_0/(tt_0 p) = \cdot 0002918 - \cdot 000035x/(1 + x).$$

Hence the effect of the water-vapour on the refractive index for air saturated at 300tt (80° F) and ordinary atmospheric pressure is about equivalent to an increase of temperature of 2° F in dry air. The temperature is therefore not determinable simply by the amount of the refraction.

Dispersion in relation to refraction. The green ray

Returning to the equation of the refraction and reminding ourselves that the deviation $\delta\theta$ depends upon $\mu_s - 1$ we must also remember that μ_s is different for light of different wave-lengths, being greater for the shorter wave-lengths. Consequently there should be some separation of colour at the earliest stage of sunrise and the last of sunset. The red colour of the sun

[1] *Q.J. Roy. Meteor. Soc.* vol. LV, 1929.

ought first to be lost at sunset and the blue or violet ought to remain as the last of the visible rays. (See G. Forbes, *The Earth, the Sun and the Moon*, p. 22, E. Benn, Ltd.)

Such a phenomenon would be very transient and therefore difficult to observe, especially as red and green are complementary colours for the human eye. But there is now a consensus of opinion that the last flash of actual sunlight in the evening and the first flash in the morning are green, and indeed a brilliant green. The subject is discussed quite effectively in a correspondence in the pages of *Nature* in 1928, which includes a letter from Prof. R. W. Wood, who expresses the opinion that the green ray is a real phenomenon, but whether it is visible or not depends on the index of refraction of the surface-air.

I have crossed the ocean some thirty times and have looked for the "ray" at every favourable opportunity, by which I mean clear sky, no haze or clouds on the horizon at sunset, and a calm sea, and yet I have observed it on only three or four occasions, and only once when it was really striking. This occasion was on an eastward trip of the *Homeric*, sailing from New York on June 6, 1925. The colour of the vanishing edge of the sun at sunset was a vivid emerald green, about the colour of a railroad signal light. On other occasions on which I have observed evidence of the phenomenon, the colour change was from red or orange to lemon yellow.

It seems possible that the determining factor is the relative temperature of the air and the ocean. Warm water and cool air would flatten the trajectory of the light rays, and cause the sun to set abnormally early. This is the type of refraction in cases of desert mirage, in which case the curvature of the rays is reversed. With cold water and warm air, on the contrary, the normal gradient of refractive index would be increased, the curvature of the rays augmented, and sunset would be delayed, giving a greater opportunity for atmospheric dispersion to come into play.

...On the day on which we observed the ray, the temperatures of air and water were practically the same at sunset. On the other three favourable evenings, on which we failed to see any trace of the phenomenon, the ocean was from twelve to fourteen degrees warmer than the air at sunset.

(R. W. Wood, 'Factors which determine the occurrence of the green ray,' *Nature*, vol. CXXI, 1928, p. 501.)

Once more therefore we get an example of the influence of the counterlapse at the surface which as we shall find intrudes itself into so many aspects of the physics of the atmosphere.

If the surface-air is exceptionally cold the index of refraction of the surface-air is high, and it may be sufficient to show the dispersion between the green and the red of the spectrum in the last rays of the sun, but if the atmosphere is near the state of convective equilibrium, the green ray is not visible.

Scintillation of the stars

Refraction by the atmosphere is also held to be responsible for the scintillation of the stars. The unmistakeable flickering has been known for ages as distinguishing the stars from the planets which shine with equally bright but steady light. The most complete explanation of scintillations is due to Respighi, 1872, who examined the star-light spectroscopically and found shadows passing over the spectrum of the stars, in normal conditions of the atmosphere from red to violet in the western stars and violet to red in the eastern. "Good de-

finition and regular movement of the bands appear to indicate the continuance of fair weather, while varying definition and irregular motion seem to imply a probable change."

The apparent lack of scintillation of a planet is probably due to the finite area of the planet's disk, as compared with the point-source of light which represents a star: the combination of all the scintillations of the points of a finite disk will make the identifications of travelling bands appropriate to any one point difficult or impossible.

D. Brunt reports that he has found evidence of scintillation being produced by oscillations in the atmosphere in the course of an investigation in which a photographic plate was exposed in a fixed direction to get a spectrum of Sirius. In the absence of any clock-drive the image of the star was expected to drift across the plate and give a spectrum of width corresponding with the time of exposure. But in fact the spectrum was intercepted periodically as if the star were alternately visible and invisible. If it might be assumed that the light from the star were deflected by a periodic change in the density and consequently in the refraction of the atmosphere, the alternation of visibility and invisibility would be explained.

The transit of rapidly moving shadows appears again in the shadow-bands which are observed to pass over the approaching edge of the totality shadow of the moon in a total solar eclipse.

With the subject of scintillation may be mentioned also the "boiling of the sun's limb," a shimmering which is often noticeable, and sometimes conspicuous, on the edge of an image of the sun thrown upon a screen by a telescope adjusted for that purpose. Attempts have been made to interpret the boiling of the limb as a weather-indicator. Like other indications which are obtained from single samples of the atmosphere, it is hard to suppose that the sample which the observer happens to have under observation contains within itself the key to the general circulation of the atmosphere and its changes.

The shape of the sky

One other phenomenon which exhibits peculiarities of appearance that may be attributed in part to refraction is the impression which an observer forms of the relative magnitude of objects in different parts of the sky. In an actual photograph, as in that of G. A. Clarke[1], there is not much difference in size of the sun or moon when it is near the horizon from what appears near the zenith. The orb is slightly elliptical by the contraction of the apparent vertical diameter, but its horizontal diameter is preserved.

The amount of contraction is rather surprising, considering that there are only 30′ of arc between the top and bottom limb of the sun or moon. It may amount to about 6′, or one-fifth of the disc. It is another example of the special effects of refraction in a long path at low altitudes.

It is the common experience of observers without instruments that the sun and moon near the horizon appear to be immense orbs compared with the

[1] *Meteorological Magazine*, vol. LVI, 1921, p. 224.

estimate formed of the size of these bodies nearer the zenith. And even in that case the personal estimate of size is exaggerated very much above what corresponds with the optical angular diameter of either sun or moon, both of which happen to be nearly 30′ of arc. If an unpractised person is asked to estimate the size of the sun or moon he will generally agree to a figure which is much too large, and which would place those bodies at a distance of about 20 metres.

Somewhat similar over-estimates are made of the angular elevation of hills or other objects on the horizon or near thereto. At the eclipse of the sun on 29 June 1927, an angular altitude of twelve degrees gave the sun a position in which it appeared quite high in the sky; and an ordinary estimate of 45°, that is halfway between the horizon and zenith, is often not more than 25° in actual altitude.

Whether these very imperfect estimates are due to some physiological or psychological influence common in humanity is a question to which no satisfactory answer can be given, but the cause, whatever it may be, is reasonably helpful in connexion with the large appearance of the rising or setting sun or moon.

The faulty power of estimate is not without its importance in meteorology, because an observer who is expected to form an estimate of the number of tenths of sky covered by cloud is hardly likely to give a correct figure if he is liable to over-estimate the height of clouds which are not far above the horizon by as much as 100 per cent. of the angular altitude. However in compensation there is this to be said, that for a hemispherical dome as of the sky, the zone which reaches from the horizon to an elevation of 30° covers one-half of the area of the hemisphere, and presumably also one-half of the area of cloud in an overcast sky.

THE EFFECT OF SOLID AND LIQUID PARTICLES

Reflexion and scattering

The color of the cloudless sky, though generally blue, may, according to circumstances, be anything within the range of the entire spectrum. At great altitudes the zenithal portions are distinctly violet, but at moderate elevations often a clear blue. With increase of the angular distance from the vertical, however, an admixture of white light soon becomes perceptible that often merges into a grayish horizon. Just after sunset and also before sunrise portions of the sky often are distinctly green, yellow, orange, or even dark red, according especially to location and to the humidity and dust-content of the atmosphere. Hence, these colors and the general appearance of the sky have rightly been used immemorially as more or less trustworthy signs of the coming weather. (W. J. Humphreys, *Physics of the Air*, p. 538.)

In the previous section we have been concerned with the optical effects of the variations of density of transparent air; other striking and interesting optical effects are caused by the small particles, solid or liquid, which are carried by the atmosphere, as distinguished from the material of which the atmosphere itself is composed. We deal first of all with the light which is "scattered" or "diffused" by reflexion from the particles. We have in mind

for the moment as the most brilliant example the "silver lining" of a cloud that is illuminated by the sun behind it, and the hardly less magnificent effect of the towering cloud of cumulo-nimbus, so splendidly white when the sun shines on it from behind the observer or from the side. In both cases the light is white, scattered or diffusely reflected. In the case of the silver lining the diffused light is the part that passes towards the observer beyond the obstacle which diffuses the light; and in the case of the sunlit cumulus the diffused light comes back towards the observer from the illuminated cloud; in that case the process comes more nearly within the original meaning of the word "reflected." The two together illustrate quite effectively what is meant by the scattering of light. It is exhibited in a less striking manner in all forms of cloud, including the nebula and the dust-haze, either of which is visible in "the sun drawing water."

Sidney Skinner points out that all objects with rough edges, of which a small circle of brown paper detached from its environment by tearing is a good example, show a brilliant "silver lining" when used at a distance to protect the eye from the rays of the sun. He has reminded me that the effect is displayed in the Alps in a very brilliant manner when the sun is just on the point of rising above a distant crest that is fringed with trees, and referred me to Tyndall's description of the phenomena. The following quotation will be a sufficient assurance to the reader that the observation is worth attention.

> You must conceive the observer placed at the foot of a hill interposed between him and the place where the sun is rising, and thus entirely in the shade; the upper margin of the mountain is covered with woods or detached trees and shrubs, which are projected as dark objects on a very bright and clear sky, except at the very place where the sun is just going to rise, for there all the trees and shrubs bordering the margin are entirely,—branches, leaves, stem and all,—of a pure and brilliant white, appearing extremely bright and luminous, although projected on a most brilliant and luminous sky, as that part of it which surrounds the sun always is. All the minutest details, leaves, twigs, etc., are most delicately preserved, and you would fancy you saw these trees and forests made of the purest silver, with all the skill of the most expert workman. The swallows and other birds flying in those particular spots appear like sparks of the most brilliant white.
>
> Neither the hour of the day nor the angle which the object makes with the observer appears to have any effect...but the extent of the field of illumination is variable, according to the distance at which the spectator is placed from it. When the object behind which the sun is just going to rise, or has just been setting, is very near, no such effect takes place.
>
> (*The Glaciers of the Alps and Mountaineering in 1861*, by John Tyndall, quoting a letter from Professor Necker to Sir David Brewster. Everyman's Library, p. 157.)

The point which deserves the reader's most careful attention is that generally the scattered light which comes from clouds is white. Some light is cut off; the intensity of daylight is reduced by an overcast sky; shadowed clouds are grey, a thunder-cloud may be almost black; but ordinarily clouds are not coloured by the scattering. Cloud-particles apparently treat all wave-lengths impartially. When clouds are coloured as in sunset-glows it is because they are illuminated by coloured light; they do not themselves initiate the colour.

It is necessary to draw attention to the fact because the theory of scattering, which is due to the late Lord Rayleigh, was developed in connexion with the explanation of the blue colour of the sky, and led to the conclusion that the part of the energy of a beam of light which would be scattered by a cloud of particles, small in comparison with the wave-length of the light, is proportional inversely to the fourth power of the wave-length. In those circumstances since the wave-length of the violet end of the spectrum is only one-half of that at the red end, the percentage of the energy of the scattered red should be only one-sixteenth part of that of the scattered violet.

The theory of which some account is given on p. 151 depends upon the assumption that the effect of the particles is to load the aether which is vibrating, so that the forces of displacement have to operate upon something equivalent to greater mass. The hypothesis is somewhat artificial but sufficient for its purpose. It should presumably take a new form if the general hypothesis of a luminiferous aether is replaced by a new form of wave-dynamics.

Lord Rayleigh turned to the molecules of the air itself as being probably the particles which caused the scattering, and from his theory it followed that the ultimate colour of the sky is blue, or possibly in the purest conditions violet, on account of the effect of the molecules of air. The explanation thus given is strongly supported by the computation by Schuster, to which we refer in chap. IV, p. 152, that the loss of energy of solar radiation through the scattering by molecules of air is sufficient to account for the actual losses which are computed from observations of solar radiation on Mount Wilson. The production of blue colour in a beam of light which traverses dust-free air has been demonstrated experimentally by the present Lord Rayleigh[1].

That clouds are generally white or grey and show no colour in spite of the proportionality of the scattering to the inverse fourth power of the wave-length is very remarkable.

The blue of the sky

The blue colour of the sky may be attributed, as we have already mentioned, to the scattering of sunlight by the ultimate molecules of air. The colour is not by any means a pure blue; violet, blue, green and yellow are all scattered. A tube pointed to a blue sky shows a white patch, not a blue one, on a screen that receives the light which passes along the tube. An artist's studio is generally arranged to be illuminated by a skylight facing towards the north, but the sense of colour in the studio is not understood to be impaired thereby when the sky is cloudless. Reds are visible as well as blues, greens or violets.

The blue colour is much stronger as seen at great altitudes than from places near sea-level, and the colour is not by any means the same over the whole sky. It is affected by the scattering from the larger particles, either solid or liquid, which are more abundant nearer the surface; hence the colour of the sky gradually changes from a deep blue at high altitudes to a greyish white, sometimes with a tinge of red or brown, along the horizon.

[1] *A Dictionary of Applied Physics*, vol. IV, Macmillan and Co. Ltd., 1923, s.v. 'Scattering of light by gases.'

Neither is the blue the same from day to day. In the clear weather of a north-westerly wind the sky at high angles of altitude is deep blue, but with an east or south-east wind it may be so pale that the blue can hardly be discerned at all. Information about the colour of the sky in different parts of the earth is rather scanty. In mid-ocean, so far as the limited experience of the writer goes, the sun does not rise or set in cerulean blue but in white or grey. Judging by casual records in travellers' tales, even in desert countries where the sun shines from its rising to its setting, the sky is only blue in the early morning and settles down to a dull brazen colourless uniformity which lasts through the day. In northern latitudes the pallid hue may be due to the condensation of water on salt or other hygroscopic nuclei at some degree of humidity below saturation and the consequent addition of white light scattered by the particles sufficient to dilute the original blue or even to overwhelm it; on occasions the eruption of a volcano may spread fine dust throughout a great part of the atmosphere and veil the customary blue to so remarkable a degree that the dust may also behave like a nebula or thin cloud (p. 330).

The relation between sky-colour and the number of dust-particles has been examined in Washington[1].

On our Lapland expedition (1927) I ascertained that polar air has a deeper (blue) coloring than sea or tropical air. Before approaching cloudiness there occurs a marked decrease in blue coloring; a lighting up of the sky caused by hydroscopic enlargement of the aerosols.

(F. Linke, *Monthly Weather Review*, vol. LVI, 1928, pp. 224–5.)

When the sky is really blue the light which comes from it is polarised. Viewed through an analyser set at right angles to the direction of the sun's rays, there is considerable polarisation with vibrations perpendicular to the direction of the sun's ray and the direction of the ray under observation. The polarisation of the light of the sky has been the subject of much investigation; it has been explained by Lord Rayleigh on the undulatory theory as a natural consequence of scattering, and the explanation has been adapted by Sir A. Schuster for the electromagnetic theory of light. Much attention has also been given to observation; polarisation is one of the regular subjects of observation at the Physikalisch-Meteorologisches Observatorium at Davos.

The fraction of the light which is polarised in the plane through the observer, the point observed and the sun, increases as the analyser (L. Weber, F. F. Martens, Cornu or Savart) is directed to points at successively greater distances from sun or counter-sun in the vertical plane through those two points. For other points observations become complicated by the distinction drawn between polarisation in the plane through sun, counter-sun and observed point which is called positive, and attributed to the scattering of direct sunlight, and polarisation in a plane at right angles to the first which is attributed to secondary scattering and is called negative.

Three neutral points where positive and negative polarisation are equal and in consequence plane polarisation is lacking are named after their discoverers, Arago, Babinet, Brewster.

[1] I. F. Hand, 'Blue sky measurements at Washington, D.C.,' *Monthly Weather Review*, vol. LVI, 1928, p. 225.

The details form a very specialised section of the physics of the atmosphere. C. Dorno refers his readers to Busch and Jensen[1]. In English the special articles in the *Encyclopaedia Britannica* or in the *Dictionary of Applied Physics* may serve the same purpose.

It has been suggested that regular measures of polarisation might furnish a means of showing changes in the atmospheric structure on which predictions of future weather might be based; but once more we must repeat that it is too much to expect to sound the whole of the atmospheric ocean, or indeed any considerable region of it, with a single plummet. So much of the action of the atmosphere depends upon space differentials.

Polarisation enters hardly at all into the considerations of energy with which in this volume we are chiefly concerned, and we may be content here as elsewhere in indicating the places where the subject is more adequately treated.

Artificial "blue sky"

Molecules are much smaller than the particles which form clouds. According to Rutherford and Geiger there are $\cdot 272 \times 10^{20}$ in a cubic centimetre. Whetham suggests that there are not more than ten million in a row of the length of a millimetre. They are certainly small compared with the wave-lengths of visible light[2] which are between ·4 micron and ·8 micron, about two thousand or a thousand to a millimetre. In chap. VIII the magnitude ·01 mm or 10 micron is assigned to cloud-particles in the atmosphere. From observations with the Owens dust-counter the particles of dust or smoke shown in the microscope are found to range in size from ·3 micron to 1·7 micron, and there is evidence of the existence of molecular aggregates, which hardly come within the meaning of the word particle, and which are less than ·2 micron. From the dimensions of Bishop's ring (see p. 84) Pernter[3] estimated that the average diameter of volcanic dust that remained suspended in the atmosphere was 1·85 micron.

The limit of size for the production of colour in the scattered light appears to lie between the limits here indicated for solid particles and for the water-drops of clouds. The range of size of the solid particles is, roughly speaking, the same as that of the wave-lengths of visible light, and it is not unreasonable to suppose that the difference represented by the fourth power of the wave-length may show itself in more conspicuous scattering of blue than of red

[1] 'Tatsachen und Theorien der atmosphärischen Polarisation,' *Jahrb. der Hamburger Wissenschaftl. Anst.* XXVIII, 1910.

[2] The following table of the diameter of molecules calculated by the kinetic theory is taken from an article on "Molecule" in the *Encyclopaedia Britannica*. The unit is believed to be cm.

	From deviations from Boyle's law	From coeff. of viscosity	From coeff. of conduction of heat	From coeff. of diffusion	Mean
Hydrogen	$2 \cdot 05 \times 10^{-8}$	$2 \cdot 05 \times 10^{-8}$	$1 \cdot 99 \times 10^{-8}$	$2 \cdot 02 \times 10^{-8}$	$2 \cdot 03 \times 10^{-8}$
Carbon monoxide	—	$2 \cdot 90 \times 10^{-8}$	$2 \cdot 74 \times 10^{-8}$	$2 \cdot 92 \times 10^{-8}$	$2 \cdot 85 \times 10^{-8}$
Nitrogen	$3 \cdot 12 \times 10^{-8}$	$2 \cdot 90 \times 10^{-8}$	$2 \cdot 74 \times 10^{-8}$	—	$2 \cdot 92 \times 10^{-8}$
Air	$2 \cdot 90 \times 10^{-8}$	$2 \cdot 86 \times 10^{-8}$	$2 \cdot 72 \times 10^{-8}$	—	$2 \cdot 83 \times 10^{-8}$
Oxygen	—	$2 \cdot 81 \times 10^{-8}$	$2 \cdot 58 \times 10^{-8}$	$2 \cdot 70 \times 10^{-8}$	$2 \cdot 70 \times 10^{-8}$
CO_2	$3 \cdot 00 \times 10^{-8}$	$3 \cdot 47 \times 10^{-8}$	$3 \cdot 58 \times 10^{-8}$	$3 \cdot 28 \times 10^{-8}$	$3 \cdot 33 \times 10^{-8}$

[3] W. J. Humphreys, *Bulletin of the Mt Weather Obs.* vol. VI, 1913, p. 9.

by particles which are not technically small compared with the wave-length; which are in fact about of a size with it. If that be so a beam of sunlight passing through any considerable thickness of lower atmosphere in which there are always dust-particles in varying numbers would emerge as a red-coloured beam because it would have lost by scattering a larger percentage of its blue components than of its red components.

The separation of red light by transmission of a beam through a medium which scatters the blue is illustrated by many beautiful experiments. John Tyndall gives the following account of one such experiment.

> I shall now seek to demonstrate in your presence, *firstly*, and in confirmation of our former experiments, that sky-blue may be produced by exceedingly minute particles of any kind of matter; *secondly*, that polarisation identical with that of the sky is produced by such particles; and *thirdly*, that matter in this fine state of division, where its particles are probably small in comparison with the height and span of a wave of light, releases itself completely from the law of Brewster; the direction of maximum polarisation being absolutely independent of the polarising angle as hitherto defined. Why this should be the case, the wave-theory of light, to make itself complete, will have subsequently to explain.
>
> Into an experimental tube I introduce a new vapour, in the manner already described, and add to it air, which has been permitted to bubble through dilute hydrochloric acid. On permitting the electric beam to play upon the mixture, for some time nothing is seen. The chemical action is doubtless progressing, and condensation is going on; but the condensing molecules have not yet coalesced to particles sufficiently large to scatter sensibly the waves of light. As before stated—and the statement rests upon an experimental basis—the particles here generated are at first so small, that their diameters do not probably exceed a millionth of an inch ($2{\cdot}5 \times 10^{-5}$mm.), while to form each of these *particles* whole crowds of *molecules* are probably aggregated. Helped by such considerations our intellectual vision plunges more profoundly into atomic nature, and shows us, among other things, how far we are from the realisation of Newton's hope that the molecules might one day be seen by microscopes. While I am speaking, you observe this delicate blue colour forming and strengthening within the experimental tube. No sky-blue could exceed it in richness and purity; but the particles which produce this colour lie wholly beyond our microscopic range. A uniform colour is here developed, which has as little breach of continuity—which yields as little evidence of the individual particles concerned in its production—as that yielded by a body whose colour is due to true molecular absorption. This blue is at first as deep and dark as the sky seen from the highest Alpine peaks, and for the same reason. But it grows gradually brighter, still maintaining its blueness, until at length a whitish tinge mingles with the pure azure; announcing that the particles are now no longer of that infinitesimal size which reflects the shortest waves alone. (Possibly a photographic impression might be taken long before the blue becomes visible, for the ultra-blue rays are first reflected.)
>
> The liquid here employed is the iodide of allyl, but I might choose any one of a dozen substances here before me to produce the effect....In all cases, where matter passes from the molecular to the massive state the transition is marked by the production of the blue. More than this:—you have seen me looking at the blue colour ...through a bit of spar. This is a Nicol's prism....The blue that I have been thus looking at is a bit of more perfect sky than the sky itself. Looking across the illuminating beam as we look across the solar rays at the sky we obtain not only partial polarisation, but *perfect* polarisation.

(John Tyndall, *Heat a Mode of Motion*, Longmans, Green and Co., London, 5th edition, 1875, p. 514.)

In water a similar colour may be observed with a solution of sodium hyposulphite from which sulphur is precipitated in particles of gradually increasing size by the addition of a small quantity of dilute hydrochloric acid. Blue colour of the same character is conspicuous in the pools from which calcium carbonate has settled in the industrial processes in which lime is used, the brilliant colour being presumably due to very small particles still suspended in the water which is otherwise perfectly clear. For the same reason almost any chalky water in sufficient depth appears tinged with blue in a white bath.

It is on the hypothesis of the scattering of the blue by small particles in a solar beam and consequent red in the transmitted light, that the red colour of the low sun may be explained. It may vary from the deepest crimson when there is a "pure" cloud of very fine dust-particles to the pale "watery" yellow of a cloud which carries something larger than dust-particles, perhaps globules either of water or water-laden nuclei. Ordinary globules of water on the other hand, which are ten times as big as dust-particles, diffuse light by reflecting it, like an ordinary spherical mirror. Each globule becomes a centre from which its share of the light is radiated and the cloud of globules forms as it were a new source of diffused light as represented in fig. 39.

The formation of water-globules seems to be independent of the solid dust-particles: the nuclei for condensation are probably quite different[1]. This hypothesis may be confirmed to some extent whenever the sun can be viewed through a surface-fog. In London, for example, where there is smoke-dust the sun always appears distinctly outlined as a red-coloured disk, but a thin cloud, or country fog, that gives a similar outline of the sun makes no colour. It seems therefore that the colour is produced by the dust-particles carried with the fog and not by the water-particles of the cloud or of the fog. The dust-particles and the fog-globules seem to be acting independently, at least in the early stages of condensation.

The sun's rays are coloured red by the more effective scattering of the components of shorter wave-length, when so far as we can tell there are no water-particles in the sky, and especially when there is a great length of atmosphere to be travelled through on account of the sun's being at or near the horizon or especially just below it. The whole sky is then suffused with red light and every object which would ordinarily appear white takes on a red colour.

Blue shadows

The complementary blue light scattered from the dust-particles is seen in any landscape which is illuminated by cross rays of the sun whenever there is a dark background of mountains or buildings to keep out the overpowering white light of the sky above the distant horizon. It furnishes in fact the justification of the blue colour with which artists suffuse the distances and the shadows of their landscapes.

[1] G. Melander, *Union Géodésique et Géophysique Internationale, Procès-verbaux des séances de la Section de Météorologie, Prague, 1927*, p. 99, Rome, 1928; R. K. Boylan, 'Atmospheric dust and condensation nuclei,' Dublin, *Proc. R. Irish Acad.* vol. XXXVII A, No. 6, 1926, pp. 58–70.

If Thibetan mountains are arid, bare and uninteresting yet with the monsoon comes the haze which transfigures plain and mountain and afterwards made Somervell despair of finding in his palette a blue of sufficient brilliance and intensity to reproduce the colour of the shadows twenty or thirty miles away.

(Sir Francis Younghusband, *The Epic of Mount Everest*, E. Arnold, 1926.)

Countries which usually have a sky of nebulous cloud must be excepted from such a description. Northern climates show a good deal of nebulous cloud and the light scattered from the northern atmosphere as shown by the "sun drawing water" is generally white or grey and not blue.

But scattered blue can be seen in the smoke from a peat fire or from a fire of wood or leaves or garden-rubbish at any time when the sunshine crosses it. At the same time, if the sun be looked at through the smoke of such fires it appears a brownish red. In like manner tobacco-smoke in sunlight with a black background is quite notably blue and the same smoke as a transparent medium shows brownish red. The like cannot be said about coal-smoke which is at best grey and sometimes looks black in any light. The difference between the two is explained by the fact that as viewed in the Owens dust-counter wood-smoke is made up of globules of tarry liquid, and coal-smoke contains a large number of particles of solid soot.

Hence we may draw the line between selective scattering with the production of colour and collective reflexion without separation of colour somewhere between the size of a water-particle, say ·01 mm, and the size of a dust or smoke-particle, say ·0005 mm.

The dissipation of energy by scattering

Scattering follows the same general law in respect of energy as absorption (see chap. IV). For any particular cloud a coefficient determines the percentage of energy which is scattered by unit thickness of the cloud and the same coefficient applies to successive layers of equal thickness according to Bouguer's law (p. 147). It is stated in chap. IV that the cloud of an overcast sky scatters about three-quarters of the incident energy. The same fraction of the remainder will be scattered by the next layer of the same thickness; only one-sixteenth can survive the second layer. Hence in the shadow of the shadow of a cloud there is very little light at all. A double layer of overcast sky leaves hardly any light for the surface.

The law of scattering is assumed to be the same whether the cause of it be molecular or particulate, and the law of scattering which Lord Rayleigh obtained for particles small compared with the wave-length is regarded as typical of all examples of scattering.

The difference of behaviour of blue light and red light in respect of scattering by particles which are actually present in the atmosphere has been very shrewdly illustrated by R. W. Wood[1] in photographs of the same landscape taken showing its ultra-violet light on the one hand and its infra-red light on the other. We reproduce (fig. 36) two photographs of San José, infra-red and

[1] *Proc. Roy. Inst.* vol. xx, 1911, p. 180.

ultra-violet, taken at Lick Observatory. In the ultra-violet the background is obscured by a palpable mist and there is far less contrast in the foreground. In the infra-red the distant landscape is obviously clear; the objects in the background are sharply defined and the foreground is luminous.

(*a*) Ultra-violet light (*b*) Infra-red light

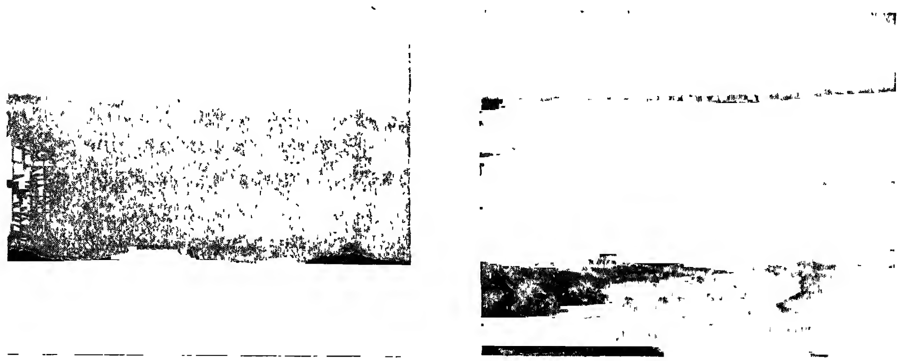


Fig. 36. San José photographed on 9 November 1924 from Mt Hamilton, 13½ miles away. The obliteration in (*a*) is due to the earth's atmosphere. (Photograph by Wright, Lick Observatory, from lantern-slides of the Royal Astronomical Society.)

Diffuse reflexion

A dissipation of energy closely akin to scattering is found in the reflexion of light from the surfaces of every kind of solid object. Under the influence of radiation of visible wave-length all objects that are not absolutely "black" diffuse the energy of the incident light in all directions. The process is known as diffuse reflexion. It may be combined with absorption in the surface-layers of the diffusely reflecting material which thereby appears coloured. This is in fact accepted, on Sir G. G. Stokes's suggestion, as the origin of the ordinary colour of natural objects. Iridescent colours are accounted for otherwise.

The diffuse reflexion of this kind combines with the regular reflexion from a transparent surface such as water, or the modification of regular reflexion which becomes scattering when drops or particles are very small, to form what is called the albedo of the earth, that is to say the reflected light by which the earth would be visible from outside and by which the separate parts are visible to us.

The measure of the illumination derived in this way from parts of the earth's surface in comparison with that which comes from an overcast sky is a subject of investigation for which a special instrument has been designed by L. F. Richardson[1].

Daylight and twilight

It is to the scattering of light that we owe the advantages of diffused daylight and twilight. For diffused daylight the light from the clear sky which is

[1] Union géodésique et géophysique internationale, Section de Météorologie, *Report on photometers for a survey of the reflectivity of the earth's surface*, 1928.

apparently blue is reinforced by the light scattered by cloud, nebula and dust, and by that which is diffusely reflected from the objects on the earth's surface.

Photometric measurements made at Mount Weather, Va. (lat. 39° 4′ N, long. 77° 54′ W, altitude 526 m) show that with a clear sky the total mid-day illumination on a horizontal surface varied from 10,000 foot-candles in June to 3600 foot-candles in January. It is less than the direct solar illumination on a normal surface from September to February, inclusive, but exceeds the latter from May to August, inclusive, for a period of from four to eight hours in the middle of the day.

The illumination on a horizontal surface from a completely overcast sky may be half as great as the total illumination with a clear sky, and is frequently one-third as great. On the other hand, during severe thunderstorms at noon in midsummer, the illumination may be reduced to less than 1 per cent. of the illumination with a clear sky.

The ratio of sky-light illumination to total illumination on a horizontal surface at noon in midsummer varies from one-third to one-tenth. In midwinter it varies from one-half to one-fifth.

When the sky is clear, the twilight illumination on a horizontal surface falls to 1 foot-candle about half an hour after sunset, or when the sun is about 6° below the horizon. (H. H. Kimball, *Monthly Weather Review*, vol. XLII, 1914, p. 650.)

Relative intensities of natural illumination in foot-candles (Kimball and Thiessen)

Zenithal sun 9600·0

Twilight for different positions of the sun's centre

Sunrise or sunset	33·0	6° below horizon	0·40
1° below horizon	30·0	7° " "	0·10
2° " "	15·0	8° " "	0·04
3° " "	7·4	8° 40′ "	0·02
4° " "	3·1	9° " "	0·015
5° " "	1·1	10° " "	0·008

End of civil twilight (6°)	0·40
Zenithal full moon	0·02
Starlight	0·00008

Twilight is the light which is scattered from the earth's atmosphere when it is illuminated by the sun below the horizon. "Astronomical twilight" is defined as the interval between times when the upper edge of the sun is on the horizon and the true position of its centre 18° below; "Civil twilight" is the interval between times when the upper edge of the sun is on the horizon and the true position of its centre 6° below.

The Aeronautical Observatory at Lindenberg has published (1929) a diagram of isopleths, too large to be reproduced, from which the duration of civil twilight (Dauer der bürgerlichen Dämmerung) for any day of the year can be read off for any latitude between 20° and 65°. For the diagram, bürgerliche Dämmerung is defined as the period during which the zenith distance of the sun is between 90° 35′ (i.e. the position at sunrise or sunset) and 96° 30′. Twilight extends to the whole of the night from May to the beginning of August in latitude 65° N, and for eight days in June a little north of 60° N. The readings of the diagram give a few more minutes to twilight than the figures in the following table for latitudes between 0° and 50° which is based on Kimball's computations. For the sake of comparison with the

table of additions to the geometrical day caused by refraction (p. 56) the duration is given in decimals of an hour.

Duration of twilight

	Civil						Astronomical					
Latitude	0°	10°	20°	30°	40°	50° N	0°	10°	20°	30°	40°	50° N
	h	h	h	h	h	h	h	h	h	h	h	h
Jan. 21	·37	·37	·38	·43	·48	·62	1·22	1·22	1·28	1·38	1·57	1·90
Feb. 21	·35	·37	·37	·40	·47	·55	1·17	1·18	1·22	1·33	1·52	1·80
Mar. 21	·35	·35	·37	·40	·45	·55	1·15	1·17	1·22	1·33	1·52	1·83
Apr. 21	·37	·37	·37	·42	·47	·58	1·18	1·20	1·27	1·40	1·65	2·13
May 21	·37	·37	·40	·45	·52	·68	1·22	1·25	1·35	1·53	1·90	2·97
June 21	·37	·38	·42	·47	·55	·73	1·25	1·30	1·40	1·62	2·05	—
July 21	·37	·37	·40	·45	·52	·68	1·22	1·25	1·35	1·53	1·90	3·00
Aug. 21	·37	·37	·37	·42	·47	·58	1·18	1·20	1·27	1·40	1·65	2·15
Sept. 21	·35	·35	·37	·40	·45	·55	1·15	1·17	1·22	1·33	1·52	1·83
Oct. 21	·35	·37	·37	·40	·47	·55	1·17	1·18	1·22	1·33	1·52	1·80
Nov. 21	·37	·37	·38	·43	·48	·62	1·22	1·22	1·28	1·40	1·58	1·92
Dec. 21	·37	·38	·40	·45	·52	·65	1·25	1·27	1·32	1·43	1·63	1·98

Twilight colours

As the sun sinks to and below the horizon during clear weather, a number of color changes occur over large portions of the sky, especially the eastern and western. The phenomena that actually occur vary greatly, but the following may be regarded as typical, especially for arid and semi-arid regions:

(a) A whitish, yellowish, or even bronze glow of 5° or 6° radius that concentrically encircles the sun as it approaches the horizon, and whose upper segment remains visible for perhaps 20 minutes after sundown.

The chief contributing factors to this glow appear to be (1) scattering, which is a maximum in the direction, forward and back, of the initial radiation, and (2) diffraction by the dust particles of the lower atmosphere. In both cases blue and violet are practically excluded, owing to the very long air-paths.

(b) A grayish blue circle that rises above the eastern horizon as the sun sinks below the western. This is merely the shadow of the earth.

(c) A purplish arch that rests on the earth shadow and gradually merges into the blue of the sky at a distance of perhaps 10°, and also fades away as the arch rises.

(Humphreys, *Physics of the Air*, p. 546; which should be consulted for further details and explanations of the phenomena.)

Sunset colours

We have already explained that the colours shown in the sky at sunset and at sunrise are due to the illumination of the clouds by light which has been coloured red by the scattering of more of the light at the blue end of the spectrum than of that at the red end, and the consequent deprivation of the rays coming from a low sun of the ordinary proportion of blue. The colour of the light with which the sky is then illuminated is very variable, depending partly on the size of the particles of dust which produce the scattering and partly on the length of the path of the light through the lower atmosphere.

One example of the action is the illumination of the snow peaks by crimson light after sunset. A notable crimson colour is seen forming the bright lining of clouds at sunset in the prairies of North America.

THE EFFECT OF WATER-DROPS

So far we have regarded solid and liquid particles as auxiliary causes of the scattering of light. We have now to consider them as opaque obstacles in the path of the rays of the sun or moon.

Diffraction of light

We are thereby brought to the most rudimentary experiments on the diffraction of light upon which the undulatory theory was constructed by Christian Huyghens, Thomas Young and Augustin Jean Fresnel, between the latter part of the seventeenth century and the middle of the nineteenth. Isaac Newton investigated the dispersion of light by a prism and other colour-phenomena, and, for reasons to which recent advances of electrical theory have brought support, declined to entertain the wave-theory. However far in the present century physical theory may have drifted away from the undulatory theory as understood in 1812, the experimental basis remains unaltered. Two narrow beams of light supplied originally from the same source "interfere" where they are superposed, that is to say they behave as though the light consisted of oscillations which can reinforce or destroy one another according as they are in the same or opposite phases. (See vol. II, p. xxxii.)

Young's fundamental experiment affords a simple illustration. A very small aperture, linear or circular, in a screen A (fig. 37), preferably linear in order to make the result more easily visible, allows a beam to pass and illuminate two similar apertures b_1, b_2 in another screen placed in front. Each of these two apertures becomes a source of light, derived originally from the common source, which illuminates a third screen C. The waves from b_1 and b_2 will arrive then simultaneously at the central point c of the screen, because the velocity of the light is the same for both and also the length of the path which it has to travel; but in all other parts of the screen the wave in one ray will lag behind that in the other, and the lag at any point will be determined by the difference in the length of the two paths tending to that point. The effect upon a beam of light, of wave-length λ, of the gradual increase of lag with distance from the middle position results in the conjunction of the waves of the two beams at successive points c_1, c_2, c_3 on the screen, and the opposition of the waves at points nearly midway between them. Consequently for that particular wave-length there will be an alternation of bright and dark lines at intervals measured from the middle bright line.

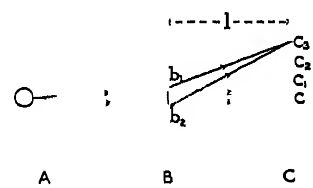


Fig. 37. "Interference" of two beams of light from a single source after passage through two small slits b_1 and b_2. In practice the distance BC is very great.

If the area of the screen under consideration be limited to an angular distance from the middle line so small that the sine of the angle may be taken as equal to its arc α on a unit circle, the distance of the first bright line from

the central line, and the separation, c, of successive bright lines on either side, will be given by the equation $c/l = \lambda/b$, where l is the distance of the receiving screen from the slits and b is the distance apart of the slits.

If the light of the incident beam is white light, and therefore includes all wave-lengths between $\lambda = \cdot 4$ micron and $\lambda = \cdot 8$ micron, the lines for the several colours will appear on the screen, violet inside and red outside, at successive distances, the second maximum of the extreme violet being just superposed on the first maximum of the extreme red; beyond that limit there will be a superposition of colour which becomes more complicated with increasing distance.

The formula may be written in the form $c/l = \lambda/b = \alpha$, and α is the angular distance of the bright line from the central line as viewed from the screen.

The process of interference which is here described is different only in experimental detail from that which has been referred to in the case of sound, as the interference of Huyghens's zones, on p. 41.

Solar and lunar coronas

The succession of bright and dark lines in Young's experiment, where the incident light is of a single colour, or the succession of colours if the light is composite, has received the name of fringes, and is due to the diffraction of the light by the central intervening obstacle. In order to see them in the actual experiment they must be received on a screen and magnified or viewed through a suitably placed eye-piece; but we can apply the reasoning to the explanation of the coloured rings which may be seen when the sun or moon is viewed through a cloud of particles not dense enough to obliterate the luminous body altogether, and uniform enough to sort out the colours by the diffraction of the beams caused by the small obstacles.

For this purpose we regard the particles as spherical and the two beams that graze a particle on opposite sides as corresponding with the two beams that come through the separate slits in Young's experiment. Every diameter of the drop furnishes a pair of interfering beams, and the arrangement of the diffracted light is consequently circular, not linear; but the separation of the bright and dark lines along any diameter follows the law which is applicable to the pair of slits.

To make the analogy quite complete we ought to have each drop surrounded by a circular screen to cut off all the light except a circular slit framing the drop. In the absence of an arrangement of that kind we have to allow for all the light which passes the drop outside the narrow circle that produces the interference; but we get over that difficulty by suggesting that there are a multitude of drops all acting together, each one blocks out something, and each one in doing so produces its own set of fringes. All the drops which we have to consider are comprised within the angular diameter of the sun or moon; they are therefore within a quarter of a degree from the centre of illumination, and if the particles are sufficiently small to give visible circles

of colour at distances of several degrees from the centre, we may consider the sets of rings of all the separate particles as approximately coincident, and the resultant set of rings as representing sums of the energy of vibration of all the separate rings. The suggestion is not exactly true, but it will serve for the immediate purpose if we allow for some confusion in the region immediately surrounding the circle of the luminary. The concurrence of the rings belonging to different particles will become more satisfactory in the regions of larger radius, in the same way as a pinhole image of the sun gets more like the sun with increasing distance.

For the first bright ring formed by a particle of diameter b we use again the approximate formula $\alpha = \lambda/b$, and now, while looking at the centre, we become conscious of a ring of particles in the position round the centre which is at the angular distance α. Every part of that ring illuminated from behind by light parallel to the illumination of the centre, will send to the observer's eye the component of its first maximum of the wave-length λ, and so on for all the particles of any other ring, so that the observer will get the impression of a series of rings surrounding the centre according to the law $\alpha = \lambda/b$, where α is now the radial angle of the select ring of particles and approximately the same as the angular diameter of the first ring of maximum illumination for a particle at the centre.

We have estimated the size of cloud-globules at 10 microns, ·01 mm. So the first ring of extreme violet for which $\lambda = \cdot 4$ micron should have an angular radius of ·04 radian or $2\frac{1}{2}°$, and the red 5°. In favourable circumstances the sequence may be reproduced though with an increasing overlapping of the colours, and it will be understood that, as representing the combination of a vast number of separate systems not exactly coincident, the definition of the rings cannot be very precise.

The result is sufficiently near to the measurements made by observers to satisfy us that the phenomena of the coronas round the sun or moon are really due to the diffraction of light by cloud-particles.

The *Meteorological Glossary* defines corona as a coloured ring, or series of coloured rings, usually about 5° radius; and the *Observer's Handbook* of the Meteorological Office, 1926 (p. 63), says:

Coronae invariably show a brownish red inner ring, which, together with the bluish-white inner field between the ring and the luminary, forms the so-called *aureole*. Frequently, indeed very frequently, the aureole alone is visible. The brownish red ring is characteristically different from the red ring of a halo; the former is distinctly brownish, especially when the aureole alone is visible, and of considerable width, whereas the latter is beautifully red and much narrower. If other colours are distinguishable, they follow the brownish-red of the aureole in the order from violet to red, whereas the red in a halo is followed by orange, yellow and green.... The size of the diameter of the ring has been erroneously suggested as a criterion for distinguishing between halos and coronae, but a corona may be quite as big as a halo. The diameter of a corona is inversely proportional to the diameter of the particles in the atmosphere by the agency of which it is formed. Bishop's ring has furnished a well-known example of such a corona.

In the year following the eruption of Krakatoa 1883 and again in 1903 after the

eruption of Mt. Pelée, a brownish red ring of over 20° radius was frequently seen with a clear sky. It was proved to be an unusually large corona.

[It is named after its first observer S. Bishop of Honolulu.]

Extending the calculation of p. 82 for an angle of 20°, and assuming that the outer edge of the ring is the position of the first red ring, the size of the particles would be about 2 microns, which suggests dust-particles rather than water-globules.

An artificial corona

The imitation of the phenomenon of the corona on an experimental scale is quite easy. It requires a small point of light surrounded by a dark background, and "dust" of very fine and very uniform particles interposed between the observer and the point of light. If an electric lamp be surrounded by a brown-paper screen in which small holes, about 2 mm diameter, are punched, and the points of light thus limited are viewed through a plate of glass which has been dusted over with the spores of lycopodium, the coronas are conspicuously visible.

The experiment furnishes in fact a method of measuring the diameter of small particles or fine fibres. This was at once recognised by Thomas Young, who based on it an instrument for measuring the diameter of wool-fibres which he called the eriometer. All that was required was to add, to the small aperture with dark surrounding, a means of measuring the angular diameter of the rings. This he provided by marking a circle of known diameter with small holes round the central aperture. Light of known wave-lengths must be used or the instrument must be calibrated by being employed upon particles or fibres of known size. The formula by which the size of the particles can be calculated from the angular radius α of a ring of known wave-length λ is that which we have used already, viz. $b = \lambda/\alpha$.

The age of coronal clouds

There is in fact an abundance of natural examples of the formation of fringes or coronal rings by diffraction in the manner described.

A very effective experiment is often provided automatically by the light of a street lamp viewed through a carriage-window which has become "steamed" by the condensation of water-vapour on the interior surface. Excellent coronas are produced in that way in the earliest stages of the process of condensation, and the question is thus raised as to whether the uniformity which the formation of coronas demonstrates is natural to the earliest part of the life-history of a cloud or is developed in later stages by the settling of the original particles.

If the particles were solid we should be disposed to argue that the gradual settling would be equivalent to the industrial process of levigation, and the cloud would develop layers, each layer containing drops increasingly more nearly uniform in size. That may perhaps account for the more gorgeous sunset colours which followed the loading of the atmosphere with dust at the

eruption of Krakatoa in 1883. But when the particles are liquid, formed by condensation and disappearing on evaporation, the situation is not controlled merely by settling. We shall point out in chap. VIII that a cloud of water-globules of approximately equal sizes is intrinsically unstable in consequence of the laws of evaporation and condensation. Consequently our second thoughts would tend in the direction of assuming uniformity in the solid or quasi-liquid nuclei which were available for the original formation of the cloud, and consequent uniformity in the original size of the drops. In that case we should find in the formation of coronas an assurance that the cloud had been recently formed rather than the suggestion of a long life-history.

Iridescent clouds

The coronas which are most frequently noticed are those which surround the moon. Those which surround the sun are less frequently visible only because the part of the sky where they are to be seen, being close to the sun, is too bright to look at without some protection. Reflexion in a mirror of black glass is one of the means of reducing the intensity of the light within manageable limits.

But the effects of diffraction are sometimes seen in the form of brilliant iridescent colour on clouds sufficiently far from the sun for the observer to be able to use his eyes without discomfort. The iridescent patches are probably fragments of coronal rings of a high order, as the angle between them and the sun may be 20° or more.

If our surmise be correct the iridescence should be regarded as indicating the recent formation of the water-drops upon which it appears.

Iridescence is due to the opacity of the particles of a cloud, not to their transparency. It may be seen for example in the artificial cloud of smoke by which sky-writing from an aeroplane is accomplished. The essential condition for its production is uniformity in the size of the particles.

Glories

The Observer's Handbook includes among the results of diffraction caused by small particles, the coloured rings which surround the head of the shadow of an observer thrown upon a fog-bank by the sun, or indeed by any bright light behind him. Such phenomena are known as the spectre of the Brocken, because they are occasionally seen there, and the coloured rings are called a "glory."

The phenomena of the shadow thrown by the sun or moon on mist are carefully described by W. Larden[1]. He points out that each point of the luminary throws on the mist a shadow with the exact outline of the observer; the shadow-throwing light from the luminary is made up of beams of parallel light inclined at a small angle. On a screen beyond the observer there will be an umbra which is shadowed by all points of the luminary, sur-

[1] *Q.J. Roy. Meteor. Soc.* vol. XXXVIII, 1912, p. 37.

rounded by a penumbra which is only partially shadowed. On a screen beyond a certain distance there will be no umbra: the whole of the area will carry some illumination. An observer looks down a conical tube of his own shadow. The umbra cannot be greater than the original object: close up it is of the same size. Hence the descriptions of gigantic shadows of the observer on the cloud or mist must be read with some caution.

"The colours are not caused by the shadow, they are due to light diffracted backwards in the same way as the corona is due to light diffracted forwards. A large outer ring, known as Ulloa's ring, which is essentially a white rainbow, is sometimes seen at the same time[1]." It is a fog-bow, that is a rainbow so close to the observer or with such large particles as to give no appreciable dispersion.

The phenomena of the Brocken spectre with its glories can sometimes be seen quite effectively on a dewy lawn by an observer in moonlight.

Dr Fujiwhara writes to the following effect:

Glory is caused by dispersion of reflecting diffraction of the sun's rays by fog-particles just as corona is caused by passing diffraction. We can see this kind of glory looking at a mass of fog from a mountain-top with the sun behind.

There is still another kind of glory which may be called *Holy Shine*. This is the luminosity surrounding the shadow of the head of the observer thrown upon dewed grass. As Lommel has shown this phenomenon is mostly due to the scattered reflexion of the sun's light at the surface of the leaf just behind the drop. The rays passing through the drop converging at a spot on the surface of the leaf and viewed by the observer through the drop give the sense of brightness. I made many experiments to ascertain the fact and also developed a theory and published them in the Journal of the Meteorological Society of Japan.

C. K. M. Douglas[2] has stated that it is usual when flying through cloud to see a corona before emergence and a glory on looking back at the cloud after emergence.

Water-drops or ice-crystals

We have treated the orderly diffraction which produces coronas as being set up by water-drops which satisfy the essential condition of uniformity at least in respect of shape if not of size. It has been supposed on account of the irregularity of shape as well as size, that ice-crystals would not show diffraction colours, or anyway would not arrange them in circles. That has given rise to the suggestion that when coronas are seen in localities where it is certain that the temperature of the air must be below the freezing-point of water, the cloud which forms the corona must consist of super-cooled water-drops, not ice[3].

There is no serious objection to such a suggestion; the existence of super-cooled water-particles is required also to explain the *ice-storms* which are common in the United States, the *verglas* of France or *glatteis* of Germany. It furnishes an interesting question in the physics of the atmosphere because we should then have to think how far a raindrop would have to be cooled

[1] *The Meteorological Observer's Handbook*, M.O. 191 (1926), p. 63.

[2] *Meteorological Magazine*, vol. LVI, 1921, p. 67.

[3] 'Coronae and iridescent clouds,' by G. C. Simpson, *Q.J. Roy. Meteor. Soc.* vol. XXXVIII, 1912, p. 291.

to make its conversion into a drop of ice complete if it once began. We come upon all sorts of possibilities such as ice with a water-centre, or water with an ice-centre, if, as would appear obvious, the latent heat of a water-drop is more than sufficient to raise the whole of the drop from the temperature at which the freezing begins to the normal freezing-point.

The other side of the question whether the particles that form coronas are necessarily water is partially answered by the observation of halo in apparently the same cloud as that which showed corona.

I find in a commonplace book the following note of an observation of 8 October 1918:

Cumuli from 3000 feet to 15,000 feet, sharply defined tops. Super-cooled water-drops were found with certainty at a temperature of 10° F (261tt) at 10,000 feet. There was also much false cirrus between 5000 and 15,000 feet, and in some places to 30,000 feet; this was in some areas mixed with super-cooled water-drops.

A halo was seen at 11,000 feet in the false cirrus close at hand and a sun-pillar was caused by ice-crystals at 8000 feet outside a shower.

The question is discussed by C. F. Brooks in the June number of the *Monthly Weather Review*, 1920.

A lunar halo and corona were visible simultaneously from Hampstead soon after 9 p.m. on December 25th [1920]. The sky was covered with a thin cirro-nebula, and no definite clouds could be seen drifting over the moon's disc. Cirro-nebula normally consists of thinly scattered ice-crystals in a layer some thousands of feet thick, not always at a great height. On this occasion the lower part of the layer evidently consisted of super-cooled water-drops. I witnessed this phenomenon once before from an aeroplane. On that occasion a solar corona was caused by a thin layer of ordinary water-drop cloud in the middle of finely scattered ice-crystals which caused a halo.

(C. K. M. Douglas, *The Meteorological Magazine*, vol. LV, 1920, p. 274.)

In Pernter's book (Part III, 1906) we find references to McConnel's observations on iridescent clouds. His conclusion was that the colours were due to diffraction by ice-needles in clouds, and they appear in flecks and not rings, because the clouds are far from the sun and do not wholly surround it. When such colours appear at 20° and more from the sun, the maxima must be of fairly high order, as for instance 5th or 6th. The smallest ice-crystals measured on Ben Nevis were 0·0074 mm. For such the 5th maximum in the ring system would appear at 23° from the centre. But the difficulty then is that the intensity should be only half a hundredth of that at the central spot.

(*A History of the Cavendish Laboratory*, Longmans, Green and Co., 1910, p. 129.)

There are many details about the phenomena attributed to diffraction which we have not referred to but which can be found in Pernter and Exner's work or other memoirs specially devoted to the subject. We have based such conclusions as we have drawn upon the simple formula of Thomas Young. H. Köhler[1], in the course of his discussion of clouds, remarks that according to Mecke the formula does not hold for small droplets of radius less than $\cdot 4\mu$ because refraction phenomena appear as well as diffraction. These are in consequence sources of error in optical measurements. Fortunately however the great majority of drops have radii to which the optical methods of measurement can be applied.

[1] *Geofysiske Publikationer*, vol. II, No. 6, Kristiania, 1922.

Refraction of light. The rainbow

When water-drops are sufficiently large to return an appreciable part of the sunlight which falls upon them by reflexion from the back surface a rainbow is formed and appears to an observer who has his back to the sun as a series of narrow circular arcs each showing the colours of an impure spectrum. The formation of the bow appears to require raindrops as distinguished from cloud-particles, as rainbows are only seen in the sunshine which follows a shower of rain.

A remarkably successful photograph by G. A. Clarke is shown in fig. 38.

Rainbows are not all the repetition of the same appearance whenever the sun shines upon falling rain. Much attention has been given by Pernter to the colour and distribution due to differences in the size of the drops. A summary is given by Whipple in the *Dictionary of Applied Physics*.

The arcs are all centred on the continuation of the line from the sun to the observer and they appear as curves in a plane at right angles to that line. The most brilliant of them is the primary bow which has an angular radius of about 42°. Not more of the arc can be seen by an observer on a level plain than corresponds with an angle of 138° from the position of the sun in a vertical plane. Hence from level ground a rainbow is invisible if the altitude of the sun is greater than 42°. When the sun is on the horizon at sunrise or sunset one half of each circular arc is visible.

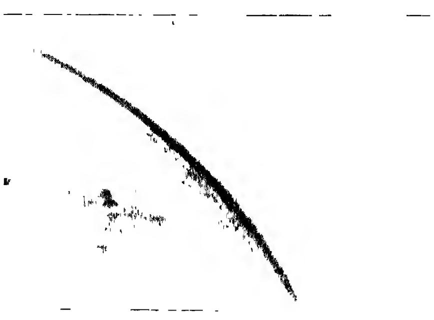


Fig. 38. Nimbus and a rainbow: 2 October 1919, 16h. A screen of rain is falling in the middle distance, and a primary rainbow, with a faint outer secondary bow, have become visible. The lighter space within the primary bow is plainly shown, and near the crest of the arch several supernumerary bands are seen. (G. A. Clarke, *Clouds*, Constable and Co., 1920.)

But if the observer is in such a position that the line of his vision at 42° below the line of the sun's ray is unobstructed except by raindrops which can perform the refraction and are also in the sunshine, the complete circle may be seen. The conditions are quite readily satisfied for example when the sun shines past an observer on board a steamer in the spray of the Niagara Falls. A complete circle can sometimes be seen there, and in other places where the corresponding conditions prevail. The order of colours in the primary bow is violet inside and red outside. The secondary bow is outside the primary, and sometimes almost as brilliant. In it the sequence of colours is opposite to that of the primary. The red is inside and the violet outside. Within the primary or outside the secondary are other arcs which are called supernumerary bows, sometimes three or four. There is a tertiary bow, which is seldom noticed, between the observer and the sun. In none of the arcs are the spectrum colours pure.

In the photograph the region within the arcs is clearly lighter than outside. In rainy districts like the west of Scotland, where they have acquired the name of sun-dogs, fugitive portions of arcs are often seen against the mountain slopes.

The primary bow is said to be formed by one reflexion at the back of the drops and entrant and emergent refractions, the secondary by two reflexions within the drop and the entrant and emergent refractions. For the supernumerary bows special portions of the reflected light are utilised. An account of their formation will be found in Humphreys's chapter on "Refraction phenomena."

The erroneous assumption that all rainbows show the same sequence of colours and have the same radius has caused the careful study of the phenomenon to be much neglected It has been shown that the colours of a rainbow as well as their extent and the position of the greatest luminosity are very variable and depend on the size of the drops producing the bow.

(*The Meteorological Observer's Handbook*, M.O. 191, 1926, p. 64.)

The formation of rainbows

The best approach to the analysis of the process of formation is a practical examination of the effect upon a sunbeam of a sphere of water. It is perhaps easier to use a sphere of glass such as the ball of a Campbell-Stokes sunshine recorder. The difference between the two does not involve any optical principle; it lies only in the difference of refractive index, 4/3 for water and 5/3 for glass. The story of the behaviour of a transparent sphere towards a sunbeam is a good illustration of the complexity of the physical problems which are presented by the atmosphere, even when the principles are simple and quite well understood.

If we take a parallel beam AA′ proceeding from a single point at a great distance (such a simplification does not happen in nature, but the sun's diameter is only half a degree of arc, and we can accept that for the moment as a point) we have first the reflexion of the rays at the outer surface of the sphere with the formation of an image at *a*, midway between the surface and the centre, not really a point-image, for obviously the ray which strikes the sphere at the 45 degree point will be sent off at right angles to its original direction. and therefore appear to come from a point on the axis which is at the distance $\frac{1}{2}r\sqrt{2}$ from the centre instead of $\frac{1}{2}r$. That does not prevent the eye appreciating a picture of a bright object by reflexion at the surface; so each raindrop in the sunshine should sparkle with an image of the sun

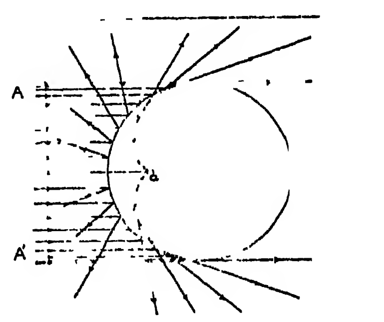


Fig. 39 The reflexion of the several parts of a beam of parallel rays of light by a sphere.

half a radius below the surface to be seen from anywhere on the same side of the drops as the sun, and a cloud of raindrops in the sun should sparkle like dewdrops on the grass. That is not the impression which an observer gets when looking at a rainbow and we must conclude that the raindrops collectively form a poor reflector compared with the smaller but much more numerous globules of a cloud.

What is left of the beam, after reflexion, enters the sphere. Each ray is subject to the law of refraction $\sin i = \mu \sin j$, where i is the angle of incidence, j the angle of refraction and μ the index of refraction. The extreme rays AX, A′X′ must be regarded as grazing the sphere without entering, and any part of the ray that gets in would have an angle of incidence 90° and an angle of refraction of 48°. This value is given by the equation $\sin 90°/\sin j = \mu$, the index of refraction of the water, that is 1·33. All the entering light is accordingly concentrated on the back surface of the sphere between Y and Y′ (fig. 40). There a considerable part gets out by refraction forming an image of the sun at Z, again not a perfect point-image but one which can be said to have a focus where indeed the card is placed in the case of a sun-recorder.

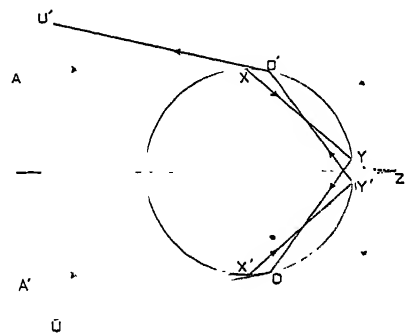


Fig. 40. The passage of a beam AA′ of parallel rays of light into a spherical drop, showing the angle over which the emergent beam UU′ is spread by refraction at entry and at emergence, and by reflexion at the back of the drop between Y and Y′.

The remainder is reflected from the surface between Y and Y′. Each ray takes a different direction on reflexion and so the reflected beam forms a fan which spreads over a wave-front extending from U to U′. The ray is deviated at entry to the extent of $i-j$, deviated again at reflexion to the extent of $180° - 2j$, and deviated again on emergence to the extent of $i-j$. So the whole deviation after one reflexion within the drop is $2(i-j) + 180° - 2j$.

There will be a second reflexion at the point where a ray after its first reflexion strikes the surface again, and again a third reflexion and so on; hence a ray which enters the sphere will be reflected over and over again, for ever, inside the sphere, and will part with some of the energy of its vibration in the form of an emergent ray at each reflexion.

So a spherical drop becomes a sort of wheel from which rays emerge after reflexion in unnumbered and indeed innumerable directions. The deviation of any ray depends on the angle of refraction j, and that again depends upon the refractive index μ. Consequently the direction of the emergent ray will be different for every different wave-length of the incident light, and the complicated fan of rays into which the incident beam has been converted will everywhere be coloured.

The angles of incidence on the inner surface are all the same for the same colour of the same ray; each of them is equal to j the angle of the first refraction. If j should happen to be exactly commensurable with 360° the same place of internal incidence will be repeated, but that could only be for one colour.

Every reflexion adds $180° - 2j$ to the deviation, and alters the position from which the refracted rays emerge, but it makes no difference to the form of the fan except that at each reflexion the amount of light is always reduced by the amount which has been refracted outwards.

We have been thinking so far of parallel rays that lie in the plane of the paper, and what has been said is equally true of the rays in any plane that passes through the centre of the sphere; so the distribution of the light which emerges after one reflexion will include that which is derived from all the rays between A and A′. The axis of the pencil, the ray that passes through the centre of the sphere itself, will be reflected back on its own path and its deviation will therefore be 180°.

The sprinkling of coloured light which the emerging beam provides is so much dispersed over the region in front of the sphere that the effect would pass unperceived like the images by external reflexion, if it were not for the fact that the emergent rays are much more congested in one part of the field than in others. The congestion takes place not far from the position of the extreme ray A, which can be identified by the following calculation:

For one reflexion the deviation D of the incident ray is given by the formula

$$D = 180° + 2i - 4j.$$

This will pass through a maximum or a minimum where $\delta D = 0$; that is where

$$di = 2dj,$$

since $\quad \sin i = \mu \sin j$, and $\mu = 4/3$, $\cos i\, di/dj = \mu \cos j$,

$\cos i = 2/3 \cos j$, whence $\cos^2 i = 7/27$,

$\cos i = \cdot 509$;

hence $\quad i = 59° 24'$ and $j = 40° 13'$.

From which we compute $\quad D = 180° - 42°$.

The position is a minimum if d^2D/di^2, which equals $\frac{3}{2} \sin i/2 \cos i$, is positive; and this will be the case because the angle i is less than a right angle.

Hence the ray of minimum deviation where it leaves the sphere is inclined at an angle of 42° to the line of the incident ray. The angle of incidence of the ray which suffers least deviation is approximately 60°.

Rays of slightly greater incidence than that which suffers minimum deviation and rays of slightly less incidence will pass very close to the minimum ray, the former crossing it, the latter not reaching it.

Here then is a congestion of rays forming a beam which strikes the drop with angles of incidence in the near neighbourhood of 60° and suffers minimum deviation. It goes by the name of the Descartes ray. If the incident light be horizontal, an eye for which the drop has an elevation of 42° will catch the congestion of rays that have the index of refraction 4/3.

The index of refraction is greater for violet light than for red, the deviation is in consequence greater for the shorter wave-length; the red ray is therefore more nearly vertical, and the drop which supplies the red colour to an observer at O must be above the one which supplies the violet (fig. 41). In other words the red arc of the primary bow is above the violet one.

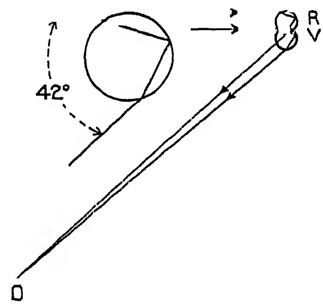


Fig. 41. The path of the Descartes ray with minimum deviation of 42°, and the relation of a drop V that shows violet to one R that shows red to an observer O on the ground.

The drops that are too low for the ray of minimum deviation to reach the eye will send to the observer rays belonging to the fringe of deviations greater than the minimum. Hence all the drops within the primary bow will send a confused mixture of rays of all colours. The effect explains why the region within the primary is brighter than that outside as shown in the photograph of fig. 38. Within that region any light which has a deviation substantially greater than the minimum finds a place.

The secondary bow is formed in like manner by the light which is reflected twice from the inner surface of the drop, and has in consequence a minimum deviation of 231°. A horizontal incident ray enters the lower part of the drop

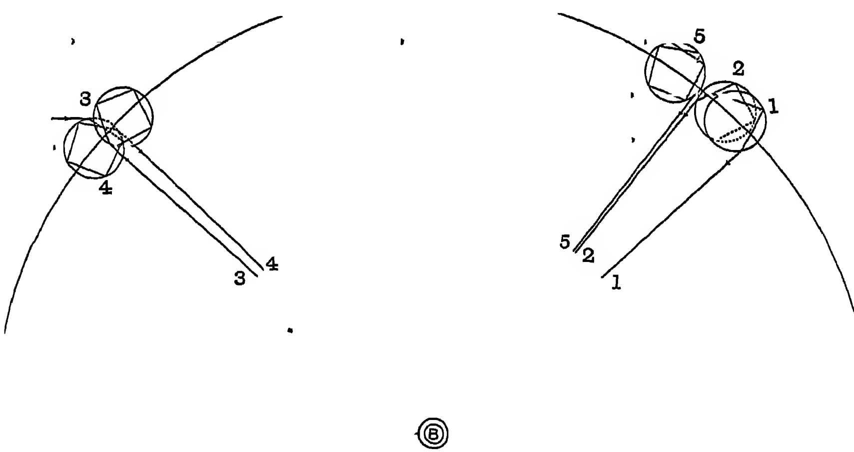


Fig. 42. Multiple reflexions in a water-drop of rays originally horizontal. Paths of the rays of minimum deviation for light which reaches an observer B after suffering one, two, three, four or five reflexions within a water-drop.

and emerges with an inclination of 51° to the horizontal. The observer must therefore look, for the secondary bow, at raindrops which are at a greater altitude than those that furnish the primary.

Other bows can be formed by additional reflexions within the drops, but the one next in order to the secondary has a deviation which requires the

observer to look for it towards the sun. In like manner the fourth bow is to be looked for against the sun's rays over the third, but the fifth bow is again in the cloud.

The tracks of the rays of minimum deviation in the first five bows are shown in fig. 42.

The rays of minimum deviation in an axial section of successive rainbows all have the same angle of incidence and cut off equal sectors from the circle; all are accordingly supplied by the small bundle of rays which forms the congestion of the first bow.

The phenomena are extremely complicated. The rainbow as a scientific instrument is not at all easy to use. It supplies an unlimited number of problems for the exercise of the ingenuity of the specialist.

The darkness of clouds

The heavy rain-clouds which generally form the background of a rainbow are dark, and indeed the darkness of an approaching storm is too well known to need description. It is characteristic of nimbus and cumulo-nimbus cloud. Darkening shadow can also be discerned on the under side of the heavier examples of summer cumulus. The other ordinary types of cloud are not suggestive of darkness.

Although darkness seems to attach to clouds from which heavy rain is falling, or will presently fall, and is therefore suggestive of large drops, rain is not so effective in restricting visibility as a cloud of finer drops. A wisp of floating cloud obliterates even the sun, but rain, even heavy rain, does not interfere to anything like the same extent with the visibility of a distant lamp.

It would appear therefore that it is not the presence of heavy raindrops in a cloud that makes the darkness underneath it, but the actual thickness of the cloud itself, above its lower surface. Great thickness is certainly a characteristic of the clouds of the cumulus and especially of the cumulo-nimbus type.

It is estimated that about 80 per cent. of the incident sunlight is scattered from an ordinary cloud of the stratus type. That would leave 20 per cent. to be disposed of otherwise, either transmitted or absorbed. Absorption probably accounts for very little, because water is very transparent for the visible rays of the sun, and the absorption of the drops is not more than that of the thickness of water which they represent. That is only a few millimetres and would not seriously diminish the luminosity of a sunbeam. So nearly the whole of the 20 per cent. may be transmitted. Very much less than that is transmitted through a dark thundercloud.

L. F. Richardson[1] has treated the question of the opacity or transparency of cloud by regarding a drop as stopping the light which falls upon it, and the transmitted light as comprising those rays which have failed to hit a drop. In that way he gets a formula of the same type as Bouguer's for the loss of light in successive layers, and the amount of light arrested becomes a measure

[1] *Proc. Roy. Soc.* A, vol. XCVI, 1919, p. 31.

of the amount of water in the cloud. In one case for example, measuring the intensity of the transmitted light with a very ingenious photometer, he computed the depth of rainfall equivalent to the water in the cloud as 24 drop-diameters.

THE EFFECT OF SNOW AND ICE-CRYSTALS

The principle of minimum deviation of light passing through a refracting medium upon which we rely for the explanation of the rainbow finds still more remarkable illustration in the atmosphere when the particles are ice-crystals instead of water-drops. And the ice-crystals have an almost unlimited capacity of combining reflexion and refraction to produce the most complicated appearances in the sky, haloes and arcs of white or coloured light, mock suns, or moons, parhelia, anthelia, paraselenae.

Our exploration of the subject will be limited to showing that the principal phenomena are within the range of explanation on accepted physical principles without invoking any new forces that might disturb the conception which we are forming of the general circulation of the atmosphere and its changes.

An experiment which is very easily made affords an excellent illustration of the use of the principle of minimum deviation in the case of haloes. The commonest form of prism, a sixty-degree prism of glass, refractive index 5/3, has an angle of minimum deviation of about 53°. Such prisms are often sold with the ends shaped to a collar terminating with a hexagon so that they can be turned round while held in front of the eye to demonstrate their effects. The prism (fig. 43) can then be held, edges vertical, suspended by a string in front of the beam of a lantern.

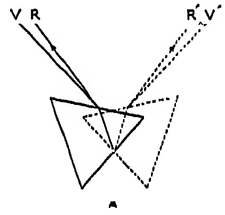


Fig. 43. A beam of light passing through a prism in its two positions of minimum deviation to form the spectra RV, R′V′ on the screen. No light that traverses the prism can reach the part of the screen between R and R′.

None of the light which goes through the prism can reach the lantern-screen with less than the minimum deviation of 53°. And if the prism be held in the position of minimum deviation the spectral colours will be shown upon the screen, the red R, R′ nearest the centre with the smallest possible deviation of any kind of light, and the other colours in succession up to the violet V, V′ deviated more and more. The spectra RV, R′V′ for the two positions of the prism are shown in fig. 43. No light at all can get to the screen between the two R's; that is dark.

If now the prism be rotated by twisting between the finger and thumb the string which holds it, the coloured beam formed by the light passing through the prism in every other position than that of minimum deviation will be somewhere to the left of R or the right of R′, it can never get between the

two. And at the same time in the actual position for minimum deviation the light will be most concentrated and most brilliant, and it will appear to stay on the screen there longer than anywhere else. So a rapid turning of the prism by the string gives an appearance on the screen of gradually increasing light from the edges to a brilliant patch at R and R′ with the spectral colours well displayed. This appearance is represented, without the colour, in fig. 44.

If now the prism be made also to oscillate in a vertical plane the light on the screen will oscillate in like manner, so that if the prism could be rotated while it is being rapidly turned there would be a lighted area of coloured circles instead of the single diametral appearance.

That is in effect a halo similar to that seen in the atmosphere as a luminous ring of 22° radius round the sun or moon, with relative darkness inside and increasingly fainter illumination outside a ring of brilliant colour.

Fig. 44. The distribution on a screen of the light of a beam which is incident upon a glass prism rotated rapidly about its length.

In the atmosphere ice-particles take the place of the 60-degree prism; with an index of refraction 4/3 the minimum deviation is 23°. The irregular distribution and confusedness of the crystals provide the distribution which in the experiment is provided by movement of the prism.

The effect of an irregular multitude of crystals can be illustrated experimentally by allowing crystals of alum to form from the drying up of a solution on a glass plate, and placing the plate in the path of a narrow parallel beam from a lantern, when a very perfect halo will be seen on the screen.

Ice-crystals are hexagonal prisms surmounted by a pyramid (hemimorphic prism) or complicated structures composed of such prisms with various modifications at the ends. The simplest are about a millimetre long. A hexagonal prism can be obtained from the triangular prism by truncating each edge parallel to the opposite face. Any pair of alternate sides will therefore act in the same way as the sides of a 60-degree prism. In the rotation of a hexagonal crystal about its long axis there would be six positions in which the prism would act as a 60-degree prism. Hence the behaviour of a cloud of ice-particles is actually represented by the triangular prism in varying positions except in so far as the index of refraction of glass is different from that for ice (see fig. 27).

The halo of 22°, the primary result of the sun shining on a cloud of ice-crystals, is shown in figs. 90 and 91 of vol. I. Fig. 91 shows also the second halo-ring, that of 46° radius, which is attributed to refraction through two faces inclined at 90°, such as may be found at the ends of hexagonal prisms whether simple or capped. "Light can enter at the end and emerge at another face or *vice versa*."

MODEL OF HALO-PHENOMENA ON A GLASS HEMISPHERE

Fig. 45 *a*. Solar phenomena on the Great Ice Barrier, 29 December 1902, from a sketch by Dr E. A. Wilson, Plate III of vol. I of the report of the *National Antarctic Expedition, 1901–4.*

The original sketch shows: (1) suffused brightness at the sun's position, (2) the halo-ring of 22° with sun-pillar beneath, mock suns on each side, tangential arc at top, coloured arcs in the lower quadrant on either side, (3) the horizontal mock sun ring without special anthelion, (4) luminous arcs, two above and two below the mock sun ring converging towards the anthelion position, (5) four mock suns, two on the outer ring of the halo of 22°, and two with luminous patches beneath them near the crossing of the 90° halo which is not otherwise indicated, (6) a zenith circle and a fragment of the halo of 46° forming a tangential arc thereto. From a comparison with fig. 45 *b* it may be suggested that the coloured arcs right and left of the sun-pillar are also fragments of the halo of 46°.

The Chinese figure at the centre of the hemisphere with its deep shadow is intended to represent the point of view of an observer. The figure has three faces, one directed towards the sun and the others towards the mock suns at 120° on either side.

An inner halo of 18° is occasionally seen and has been photographed by C. J. P. Cave[1].

The details of halo-phenomena have been very fully discussed by Bravais, Besson, Dobrowolski, Pernter and others in the works referred to in the bibliography. The diagrams representing them are in some cases very complicated, and difficult to represent in book-illustrations on account of the many different planes in which the various rings are formed. We give here two examples (figs. 45 *a* and *b*) which have been photographed from drawings prepared on the interior surface of hemispherical glass covers, and which will be sufficient to indicate by the legend the chief elements of the phenomena.

The circles formed by refracted light show colour with red inside merging to green or to a dark mixture at the outer edge. The colours are very impure and are not generally distinguishable in lunar haloes. Colours in bright solar haloes appear not only in the circle of the halo but in tangential arcs as well.

[1] *Nature*, vol. CXVII, 1926, p. 791.

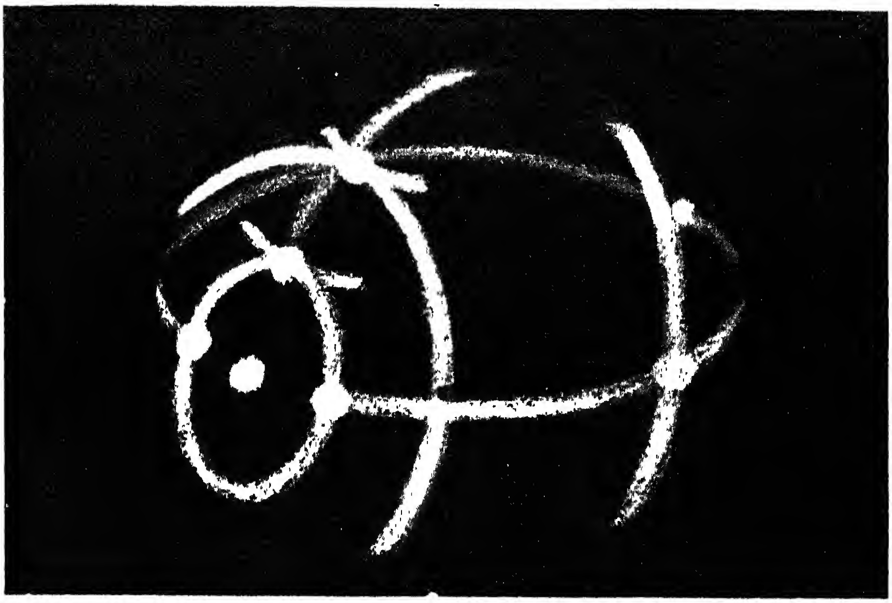


Fig. 45 *b*. The Dantzig phenomenon of 20 February 1663.

The model shows: (1) the sun's position and the horizontal mock sun ring with anthelion and mock suns where the ring crosses the 22° halo and the 90° halo, (2) the 22° halo with tangential arc at the top, (3) the 46° halo also with tangential arc, and (4) portions of the 90° halo.

Besides the refraction of light through the ice-crystals there is reflexion from the surface of the crystals which may be reduced to order by the manner in which ice-crystals of different shapes arrange themselves as they float or fall. The formation of rings, arcs, pillars or luminous patches by reflexion may be illustrated by the light on the sea formed by the rays of the sun or moon shining on the irregular rippled surface of water. If the water were perfectly smooth an observer would see only a definite image of the sun or moon reflected in the plane surface, but when the surface is disturbed irregularly rays are sent to the observer quite transiently by reflexion from every part of the rippled surface that happens at the moment to be inclined at a suitable angle to form an image visible by the observer. Each observer makes his own selection of suitable reflectors but all see a very luminous patch round the position of the image in plane reflexion. With the smaller reflecting surfaces of ice-crystals, and their great multiplicity, the luminosity is less fluctuating than that produced by water-ripples but may be more widely distributed.

The commonest examples are the sun-pillar, a column of light which extends upward or downward from the sun, and the mock sun ring, a horizontal ring of light extending all round the horizon from the sun. In this ring the concentration of light by refraction produces mock suns—intense luminescence—at the point opposite the sun, *anthelion*, or where the refraction rings cross the horizontal band, *parhelia*. The corresponding appearances for the moon are called mock moons, or *paraselenae*.

The remarkable horizontal ring all round the horizon at the same altitude as the sun can be explained in this way. It will easily be understood that if the sun shone upon a plane vertical mirror the image which an observer would see in it would be at the same apparent elevation as the sun itself; and this would be the case at whatever position round the observer the vertical mirror might be placed, provided it were turned so that the image of the sun were visible to the observer if he turned himself

in the right direction. Hence we may conclude that an observer looking round him will see an image of the sun at the same angular elevation as the sun itself in every ice-crystal, within a certain angular band, that has a vertical face turned in the proper direction. The angular band will be at least as large as the angular diameter of the sun; all suitable crystals at the proper elevation will contribute to the effect.

SNOW-CRYSTALS

Fig. 46*a*. Photographs of snow-crystals from the collection of W. A. Bentley. The white strips under the several photographs represent in each case a millimetre. (*Monthly Weather Review*, *Washington*, November 1924.)

In describing the formation of haloes we have spoken of ice-crystals as the particles which refract or reflect the light to the observer. We have had in mind such particles as might be found in cirrus cloud or in the very fine snow-dust of a mountain-summit or an Antarctic blizzard; but the ice-particles have the faculty of developing into snow-flakes if condensed water is available. We pass therefore to the consideration of snow-flakes or snow-crystals.

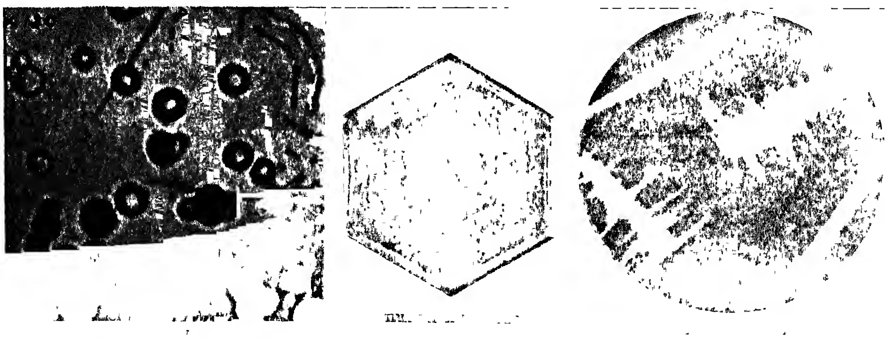


Fig. 46 *b*. Photographs of snow-crystals from the collection of W. A. Bentley. On the left nuclei of ice in water-drops (the strip below represents one-tenth of a millimetre). In the middle a single plate and on the right complicated structures; in both cases the white strip represents a millimetre.

"Among the most amazing and puzzling phenomena occurring in cloudland during the winter time is this, that the tiny cloud-droplets, $\frac{1}{500}$ to $\frac{1}{5000}$ of an inch in diameter, often retain their fluidity during zero weather, when greatly undercooled. More remarkable still is the discovery by the writer during recent years that there are times when these tiny fluid cloud-droplets have, imbedded within them, solid crystalline nuclei of hexagonal form" [as shown in the figure on the left].

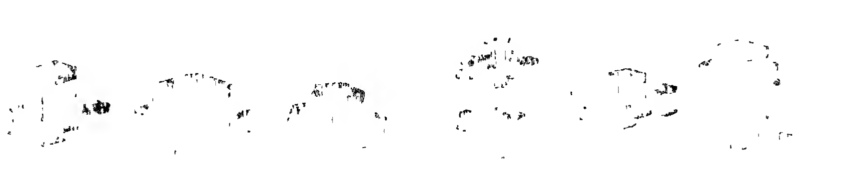


Fig. 47. Stud-shaped snow-crystals, combinations of columns and plates. From the *Classification of Snow-crystals* by J. M. Pernter, *Meteorologische Optik*, original issue, p. 285.

Snow-crystals[1], simple or composite, are of an infinite variety of shapes and sizes; many have been photographed and studied by W. A. Bentley, of Jericho, Vt., who has recently published an account of "40 years' study of snow-crystals" from the illustrations of which we have selected those which are reproduced in figs. 46 *a* and *b*. To each of them we have added a strip to indicate the size of the crystal. It will be noticed in fig. 46 *a* that the sizes vary from a small fraction of a millimetre to one and a half millimetres, and

[1] G. Stüve has recently published a collection of photographs in a paper entitled 'Die Entstehung des Schnees' in the Hergesell-Festband, Bd. xv, *Beiträge zur Physik der freien Atmosphäre*, 1929.

7-2

the thickness must be of the order of half a millimetre at least. It may perhaps be assumed that the smallest sizes are those which occur in cirro-stratus cloud of the higher part of the stratosphere which forms haloes.

Growths at right angles to the main face of the crystal are shown in fig. 47, which is taken from the original edition of Pernter's work. A case was once reported to the author of crystals in Canada which were keeled. With such a great variety of forms, all kinds of results of reflexion are possible.

OTHER OPTICAL PHENOMENA

Besides the optical phenomena which have been discussed in this chapter, there are other appearances in the atmosphere which an observer may note

SKETCHES OF THE AURORA IN THE ANTARCTIC

Fig. 48. (*a*) Auroral streamers 9 April 1902, 2h 25m a.m. (*b*) Corona, 8 April 1903, 2h a.m. (*c*) Corona, 31 May 1903, 4h p.m. (*d*) Double auroral arc, vertical rays in upper arc, 29 August 1902, 2h a.m. (*e*) Low auroral arc showing above hills, 3 June 1903, noon.

From drawings made by Dr E. A. Wilson and reproduced in *National Antarctic Expedition, 1901–4, Physical Observations*, London, 1908.

*** The word *corona* is used in two quite different senses in this chapter.

on occasions. Among them the aurora polaris, and the zodiacal light. The latter may be disposed of at once by saying that it belongs to the sun and not to the earth, and is supposed to be due to a cloud of particles forming an elliptical shape round the sun, and extending along the zodiac; it is only visible when the sun itself is below the horizon and requires a very clear atmosphere for it to be seen at all. It is more frequently seen in tropical countries than elsewhere, and is seen sufficiently often for it to have acquired an international symbol . Aurora on the other hand belongs to the earth, though the light is attributed to electrical action due to particles sent out from the sun. The appearance of aurora is sometimes accompanied by magnetic storms and is most frequently observed in the regions of the terrestrial magnetic poles, as set out in chap. II of vol. II.

PHOTOGRAPHS OF THE AURORA BOREALIS

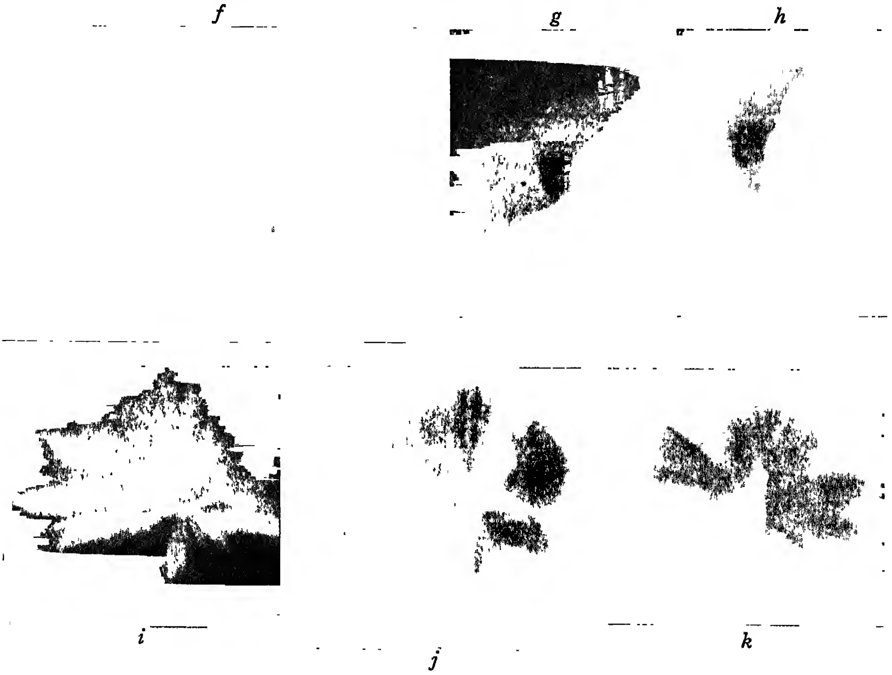


Fig. 48. (*f*) "Belle aurore boréale dans le nord photographiée à l'Observatoire de Kristiania, le 13 octobre 1916, à 10h 34m, T.M.E.C.
Pose 3 secs. La partie la plus lumineuse à droite était d'une couleur rose très belle; le reste était vert-jaune. Les étoiles ζ et η Ursae majoris se voient au-dessus de l'aurore." (Carl Störmer, 'Notes relatives aux aurores boréales,' *Geofysiske Publikationer*, vol. II, No. 8, Kristiania, 1922, Pl. III.)

(*g*) and (*h*) Carl Störmer, 'Rapport sur une expédition d'aurores boréales à Bossekop et Store Korsnes pendant le printemps de l'année 1913,' *Geofysiske Publikationer*, vol. I, No. 5, Kristiania, 1921, Pl. XXXI.

(*i*), (*j*), (*k*) Carl Störmer, 'Résultats des mesures photogrammétriques des aurores boréales observées dans la Norvège méridionale de 1911 à 1922,' *Geofysiske Publikasjoner*, vol. IV, No. 7, Oslo, 1926, Pl. XII.

Aurora is studied as an electro-magnetic phenomenon rather than a meteorological one. It is however desirable for an observer to recognise the phenomenon when it occurs. Observation is generally accompanied by sketches: we give a reproduction (fig. 48 *a* to *e*) of some made in the Antarctic by E. A. Wilson in 1902–3. Since that time a large number of photographs have been obtained by Birkeland, Störmer and Vegard, from which we select a few (fig. 48 *f* to *k*) to illustrate the more conspicuous features, and the difference of their appearance from those of rainbow, corona or halo.

Until lately it was assumed that auroral light was only visible on the occasions when there were arcs or curtains seen in the sky, but recently the spectroscope pointed at the sky has shown a green line which is characteristic of the aurora to be visible on any clear night. The subject has been primarily studied by the present Lord Rayleigh, and has added importance to the identification of the physical cause of the green ray (vol. II, pp. 27, 37) which appears now to be regarded as due to an electrical discharge through a mixture depending upon oxygen, within the province of our ordinary atmosphere.

De l'ensemble de ces travaux, McLennan conclut actuellement que d'après le spectre de l'aurore, l'oxygène et l'azote doivent exister dans la haute atmosphère, alors que ce spectre ne donne aucune indication sur la présence de l'hydrogène, et ne nécessite pas non plus la présence d'hélium. Ces résultats n'appuient donc pas l'idée déduite de l'application de la loi de Laplace, c'est-à-dire l'hypothèse de la prépondérance de gaz légers dans la haute atmosphère.

(Ch. Maurain, 'La physique du globe et ses applications,' *Revue scientifique*, 28 juillet 1928.)

CHAPTER IV

RADIATION AND ITS PROBLEMS

Normal measure of solar constant 135 kilowatts per square dekametre, 1·932 calories per square centimetre per minute. "Apparently subject to variations, usually within the range of 7 per cent."
1 g cal per cm^2 per min = 69·7 kw per $(10\ m)^2 = 6{\cdot}97 \times 10^5$ c, g, s.
1 g cal per cm^2 per day = ·0484 kw per $(10\ m)^2 = 4{\cdot}84 \times 10^2$ c, g, s.
1 milliwatt per cm^2 = 1 kw per $(10\ m)^2$ = 10 watts per sq. metre $= 10^4$ c, g, s.
100 kw per $(10\ m)^2$ = 1·43 g cal per cm^2 per min $= 10^6$ c, g, s.
1 g cal per cm^2 = 1·161 kw-hr per $(10\ m)^2$ = 4·18 joules per $cm^2 = 4{\cdot}18 \times 10^7$ c, g, s.
1 joule per cm^2 = 0·278 kw-hr per $(10\ m)^2 = 10^7$ c, g, s.
Latent heat of water 79·77 cal = $3{\cdot}33 \times 10^9$ ergs.
Latent heat of steam at 273tt 597 cal, $2{\cdot}495 \times 10^{10}$ ergs.
Latent heat of steam at 373tt 539 cal, $2{\cdot}252 \times 10^{10}$ ergs.
A beam of strong sunshine upon a ten-metre square [100 kw/$(10\ m)^2$] supplies energy at the rate of 10^{12} ergs per second. That is equivalent to a ten-metre water-fall of 3670 tons per hour,
or to heat sufficient to raise the temperature of a kilometre column of air 2·9tt per hour,
or the temperature of a slab of water one metre thick through ·86tt per hour,
or to evaporate a layer of water ·144 cm thick every hour,
or to melt a slab of ice 1·1 cm thick every hour,
or to make an air-blast 16 metres thick across the $(10\ m)^2$ area with the velocity of 10 m/sec,
or to produce a normal lightning flash every day.

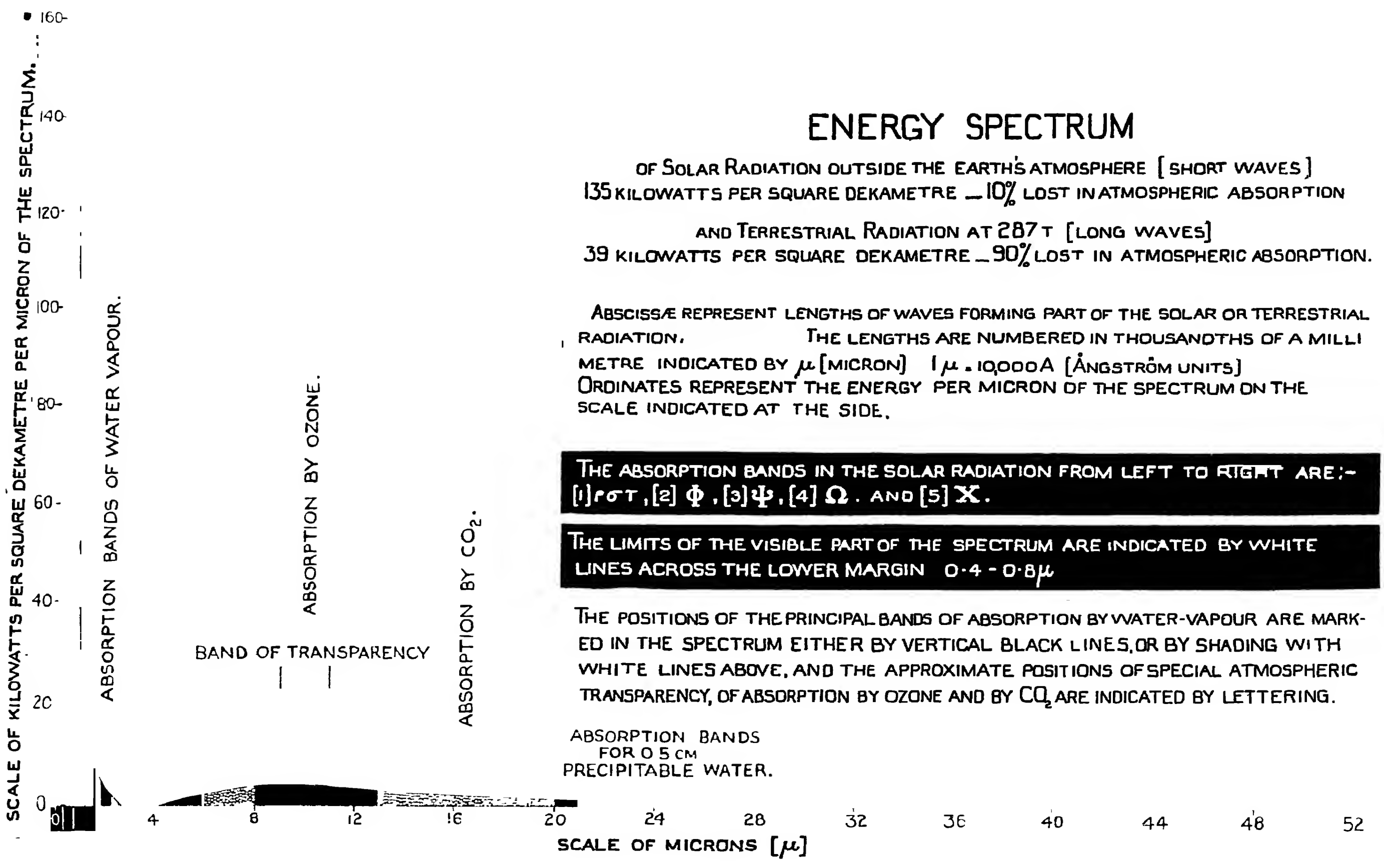
THE ATMOSPHERE AS A NATURAL ENGINE

FROM the consideration of wave-motion as illustrated by optical phenomena which have little or no influence upon the transformations of energy in the atmosphere we pass to the measurement and disposal of the energy which is associated with the wave-motion represented by radiation from the sun and sky and from the earth.

We regard weather as a sequence of incidents in the working of a vast natural engine. In its complexities the atmosphere, a heterogeneous mixture of air and steam, is at once the working substance of the natural engine and the cylinder or environment in which it works. We think of the sun as the furnace and sunbeams as the mode or agency for the conveyance of energy from the furnace to the working substance, either directly by atmospheric absorption or indirectly through the intermediary of the surface of the earth or sea. Void space itself, with its unlimited power of receiving the energy of all kinds of radiation and making no return, if we disregard the cosmic rays which are the subject of investigation by R. A. Millikan, is a very perfect alternative for the water which circulates in the condenser of an engine; terrestrial radiation is the mode or agency by which energy is conveyed into space directly from the atmosphere or through the atmosphere from the boiler (the surface of land and sea), which lies beneath it. It is customary to speak of terrestrial radiation as long-wave radiation. On the other hand the radiation received from the sun is generally spoken of as short-wave radiation. The difference between these two conceptions will be understood from the diagram of fig. 100 of vol. I which comes under reference so often that it may be repeated overleaf (fig. 49).

Falling on the earth's surface at the base of the atmosphere, the energy received from the furnace may devote itself to raising the temperature, or converting ice into water, water into steam or ice into steam, or express itself in the kinetic energy of wind.

FIG. 49. SOLAR RADIATION AND TERRESTRIAL RADIATION—RELATION OF ENERGY TO WAVE-LENGTH



The result of meteorological observations as set out in vol. II leads us to conclude that the atmosphere as a whole is not becoming permanently warmer or cooler, nor moister nor less moist; nor is the atmospheric circulation becoming permanently more vigorous or less vigorous; in other words, taken altogether, the atmosphere is making no permanent accumulation or loss of energy, thermal or kinetic.

The heat-balance

As a representation of the mode of treatment of the subject from the point of view of the working of an engine which returns to space the whole of the energy which it receives from the sun, and which depends consequently upon the conversion of "short-wave" radiation from the sun first into heat and then into "long-wave" radiation of earth, sea and air, we give a general balance-sheet of the energy involved in the process, based upon the computations of W. H. Dines[1] and represented by him in a diagram which is reproduced in slightly modified form as fig. 50.

The figures which are inserted in the balance-sheet are taken from Mr Dines's paper, converting only his figures for the day's total in gramme calories per square centimetre to kilowatt-hours per square dekametre of the earth's (horizontal) surface. Mr Dines supports his figures by consideration of the available data. In the later sections of this chapter we shall consider the results of recent measurements of some of the items, but we shall not discuss their relation to Mr Dines's figures. We are using the figures in order to represent to the reader the unavoidable complexity of the general problem of solar and terrestrial radiation. We need only remark that after a separate analysis of the debit and credit sides of the atmosphere's accounts with quite independent data Dr G. C. Simpson has satisfied himself that the accounts balance with a difference of two per cent. not merely for the whole of a normal year but separately for each month of it.

Mr Dines's figures are relied upon in order to give the reader an idea of the order of magnitude of the several items. We would not advise him to demand a justification of the details until he has made himself familiar with the information which follows. If then he is disposed to make a balance-sheet for himself we shall have achieved our purpose, and we will not spoil his enjoyment by anticipating the result.

Transactions between the surface of the earth or sea and the subjacent layers are not brought to account; they also may be regarded as being in balance, but we note a remark of Nansen's[2] about the persistent flow of water in the rivers of Greenland which come from the "inland ice" even during the winter. "The consequence is, in the lower layers of the great ice-sheet, melting must go on independently of the temperature of the surface." From this it may be inferred that some of the earth's heat is used in winter, and there is no definite evidence that it is restored in summer.

[1] 'The heat-balance of the atmosphere,' *Q.J. Roy. Meteor. Soc.* vol. XLIII, 1917, p. 151.
[2] *The First Crossing of Greenland*, Longmans, Green and Co., 1919, p. 438.

From measurements of thermal conductivity in relation to the drift of the ship *Maud*, 1922–25, the late Finn Malmgren[1] estimated the amount of heat passing from sea-water to air in the Arctic regions at 6800 g cal/cm² per annum. The supply in the cold months September to April amounted to 7670 g cal/cm², sufficient to melt ice 96 cm thick. The corresponding rate per day is 317,000 g cal/m², sufficient to raise the temperature of the lowest 150 m of air by 6·9° C.

The balance-sheet here presented is drawn for energy expressed as solar or terrestrial radiation with an allowance for heat absorbed in evaporation or conveyed by conduction at the surface. Nothing is included on account of what used to be known as the secular cooling of the earth when physicists felt justified

THE HEAT-BALANCE OF THE ATMOSPHERE

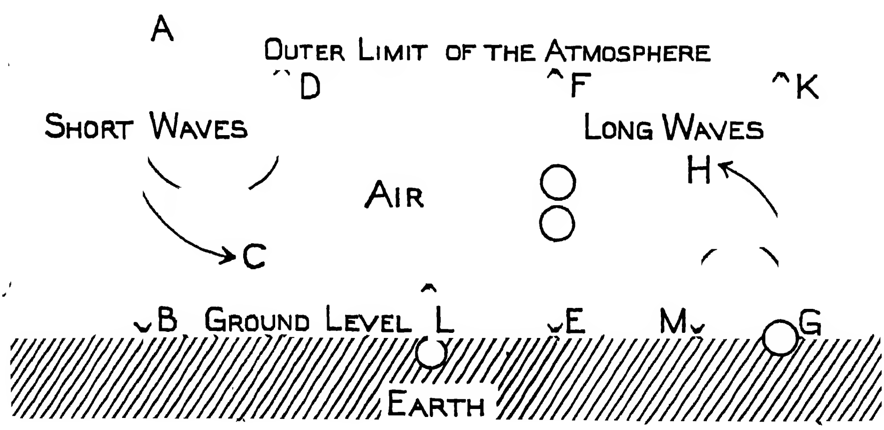


Fig. 50. W. H. Dines's scheme of transference of energy between the sun and earth and space.

in computing the age of the earth from the distribution of temperature beneath the surface; nor is anything included on account of the heating effect of radioactivity which is contingent upon the spontaneous transmutation of certain minerals. That also, perhaps properly, can be regarded as a secular effect so slow that its influence upon the sequence of weather would not be noticed within the lifetime of an ordinary observer.

A balance-sheet of this kind cannot be held to apply to any individual locality, it must be taken as representing average conditions for the whole earth. At a selected station on land or sea the outgoing radiation will depend upon the temperature of the surface and upon the composition and temperature of the air above it, which are not controlled exclusively by the amount of radiation received in the same locality.

[1] 'On the properties of sea-ice,' *Scientific results of the Norwegian North Polar expedition with the 'Maud,' 1918–25*, vol. 1, No. 5, Bergen, Griegs Boktrykkeri, 1927, pp. 64–6.

The actual figures of account for any locality would have to make allowance for the heat-transference by the movement of water and air, nothing less indeed than the general circulation of the atmosphere, and in the absence of particulars of those items the account would not balance.

Terrestrial meteorology in account with the solar system for a normal day's work in radiation

Kilowatt-hours per square dekametre of horizontal surface

I. *Sun and space*

Meteorology, *Dr.*		*Cr.*	
(A) to the sun for short-wave radiation	840	(D) by short-wave radiation reflected or dissipated by air or earth (albedo)	420
["The intensity of the solar constant is taken as 2 g cal/(cm² min), the amount received by the earth per day is $2 \times 24 \times 60 \times \pi O^2$ and this is spread over a surface of $4\pi O^2$, hence $A = 720$ [g cal/cm²]. This is capable of warming the atmosphere 3tt per day"]		(K) by long waves from earth transmitted through the atmosphere	90
		(F) by long waves radiated from the atmosphere	330
	840		840

II. *The Earth* (*land and water*)

Dr.		*Cr.*	
(B) to the sun for short waves ...	350	(G) by radiation of long waves ...	580
(E) to the atmosphere for long waves	400	(L) by transference of heat by conduction and evaporation ...	240
(M) to the atmosphere for reflexion of its own long waves ...	70		
	820		820

III. *The Atmosphere*

Dr.		*Cr.*	
(A) to the sun for short waves ...	840	(B) by short waves transmitted to the earth (direct or scattered)	350
(G) to the earth for long waves ...	580	(D) by short waves reflected (albedo)	420
(L) to the earth for heat conveyed by conduction or evaporation	240	(E) by long waves radiated to the earth	400
		(F) by long waves radiated to space	330
		(K) by long waves from earth transmitted	90
		(M) by long waves returned to earth by reflexion and scattering	70
	1660		1660

N.B. Although it merely passes through the medium, radiation which is *transmitted* is included in the balance-sheet because the composition or character of the radiation may be altered in transmission.

The radiation which is absorbed by the atmosphere and which confers upon it the radiating power expressed as E and F is made up of two parts, namely H the excess of G (the receipt from the earth) over K + M; and C which is the excess of A received from the sun over the albedo D and the transmission B.

It may be difficult to give a categorical demonstration of the ten items of the balance-sheet; but we accept it in principle when we regard the sequence of weather at the earth's surface as representing fluctuations above and below a recoverable mean value; and since the atmospheric engine is constantly being supplied with energy from the sun, and any kinetic effect is only temporary, the conditions of working are those of a complex engine in which the whole of the heat passes within a limited time from the furnace to the condenser; on the way it may produce kinetic or dynamical effects within the atmosphere itself; but the energy displayed in that way is gradually reconverted into heat. The only irreversible change during the process is the transformation of the energy from short-wave radiation, as received, into the long-wave radiation as it passes back again into space.

A daily balance-sheet for a surface of water, Lake Vassijaure, from observations extending over ten days, 21–30 August 1905, is given by A. Ångström[1].

Water surface in account with sun and sky

Dr.	$\frac{\text{g cal}}{\text{cm}^2}$	kw-hr $(10\text{ m})^2$	*Cr.*	$\frac{\text{g cal}}{\text{cm}^2}$	kw-hr $(10\text{ m})^2$
To sun and sky... ...	353	410	Returned on reflexion ...	28	33
			Long-wave radiation from water	207	240
			Reserve by evaporation...	118	137
	353	410		353	410

In this chapter we propose to examine the processes of supply of heat to the earth and its removal, and to summarise the information thereupon which has been accumulated.

The process is very much involved because a sunbeam is itself a complex of rays comprising all the wave-lengths which constitute the solar spectrum. They vary from ultra-violet to infra-red, about seven octaves of the diagram of p. xxviii of vol. II, and the behaviour of the radiation in respect of scattering, absorption and reflexion is different according to the wave-length.

RADIATION AND ITS LAWS

In the preceding section radiation has been discussed and a balance-sheet arrived at with the implicit assurance that the reader is familiar with the physical ideas which are associated with the word radiation. The conception of radiation however covers so vast a number of ideas that we may be excused for reminding the reader of those which are most pertinent to the study of weather.

In the first place by radiation is understood the automatic transference of energy from one body of material of any form, whether solid, liquid or gaseous or a combination of all three, to any and every other body which can be reached by straight lines. The transference takes place, so all the available

[1] 'Applications of heat-radiation measurements to the problems of the evaporation from lakes and the heat convection at their surfaces,' *Geografiska Annaler*, Bd. II, Stockholm, 1920, H. III, pp. 237–52.

information agrees, by some process strictly and perfectly analogous to wave-motion. Illustrations of all the properties of wave-motion which we have cited in the three preceding chapters can be given by experiments on radiation, and indeed wave-motion can be more perfectly illustrated by such experiments than by any other.

What it is which is endowed with this property of transmitting radiation, which is set in oscillation by every material object in contact with it, cannot now be so easily explained as it might have been fifty years ago. At that time physicists were agreed that space and all the material substances within it were occupied or permeated by an undulatory, luminiferous aether which had the nature of a perfectly elastic solid. We could have distinguished then between transparency and opacity. But now we are much more in the dark. The old question of the guides to science, "Why can we see through a window and not through a door?" with its answer, "Because the window is transparent and the door is not," has ceased to satisfy even the people who write books for children. Electrical science has brought to knowledge a whole range of possible oscillations which differ only in the wave-length from those which correspond with light and radiant heat. The range known to modern science is set out in fig. ii of vol. II. For many of these new wave-lengths the window and the door may be equally transparent, or the order of transparency may even be reversed if the wave-length and the material of the window or the door are skilfully chosen.

We are accustomed to think of radiation as coming across the space between us and the sun, or that between us and the fire, or even from a hot-water pipe, and further we can feel the cooling effect upon our persons of a cold environment even if the air which surrounds us is not itself cold. The warmth of radiation in the sun or the chill of exposure at the close of a clear day are among the commonest of commonplace experiences. These exchanges of energy between one body and another "within sight" are certainly associated with differences of temperature; modern experience tells us that every body radiates energy by wave-motion to every other body "within sight" at a rate of delivery which depends upon the temperature of the radiator and the nature of its surface. The radiation travels with the velocity of light if there is no material medium between the radiator and its object; when there is an intervening medium such as air or water the velocity of travel is reduced in proportion to the index of refraction of the medium, and some of the energy of the beam is lost in transmission, partly by reflexion if there is a reflecting surface in the path, and partly by absorption in the transmitting medium itself.

Whether we gain or lose warmth by the exchange is only a question of whether our temperature is lower or higher than that of the bodies which are visible in our environment—they need not indeed be actually visible, only potentially so; we might see them if their temperature were high enough for the part of the radiation which they send us to be within the limit of visible wave-lengths, i.e. wave-lengths to which the human eye is sensitive. It is

now recognised that every body everywhere all over the earth and throughout the universe is constantly sending out energy by waves that originate from its surface or beneath the surface. This radiation is a mode of automatic communication between every material body in the universe and every other.

Universal radiation

All the world is familiar with universal gravitation, the laws of which were explored by Newton. By the operation of gravity between two bodies, wherever in the universe they may be, there is a stress which is radial, that is to say it operates also in straight lines like radiation, and its intensity follows the common law of illumination, the law of inverse square of the distance. Radiation is equally universal, but its manifestations depend not upon mass but upon surface and temperature; in gravitation there is nothing quite analogous to a glass fire-screen, yet it is possible to say that gravity must be a form of radiation with more assurance than it was formerly, now that such an enormous variety of phenomena has been connoted by the term.

The dependence upon temperature is very remarkable; the rate at which any body transfers its energy by radiation depends upon its temperature measured not from the freezing-point of water or of mercury or of anything else, but from a point which is called the absolute zero, and the temperature so measured is the absolute temperature described on p. xix of vol. II. Within the practical limits of meteorological measurement the absolute temperature is expressed in this book by what we have called the tercentesimal temperature. It is in effect the temperature which is concerned in the gaseous laws. Its increase is proportional to the corresponding increase in volume of air at constant pressure, or to the increase of pressure of air at constant volume. The difference between the temperature which expresses that idea and the absolute temperature as rigorously understood by physicists is an uncertain fraction of a centigrade degree, about a tenth.

In practice absolute temperature is regarded by some meteorologists as an unreasonable innovation, a thing which no ordinary mortal should be asked or expected to comprehend. It is difficult to understand that attitude on the part of scientific authority. The idea of absolute temperature is inherent in every material object that exists in the universe, and has been so from the beginning of creation. Not only every man or animal, but every thing animate or inanimate adapts its behaviour according to its absolute temperature. To ignore that fact is to ignore one of the foundations of the physical universe.

Of course, since everything is radiating, it is the difference of temperature between two bodies that determines which of the two gains energy or loses it during the action. So if it is only a question of gaining or losing without inquiring "How much?" any scale of temperature is as good as another; but in meteorology inevitably we must want to know how much energy we are gaining from the sun and how much we are losing to the air or to space; to estimate the amount of gain and loss to make such a balance-sheet as we have given on p. 107, a knowledge of absolute temperature is indispensable. No

apology is needed for asking any student of meteorology, who feels alienated by the use of measures of temperature in the absolute or tercentesimal scale because he is not familiar with it, to seek an early opportunity of acquiring the necessary familiarity with its use.

The statements of distinguished men of science upon this subject appear sometimes to be irreconcilable:

> Heat is the motion of the atoms or molecules of a substance, and temperature which indicates the degree of heat is a way of stating how fast these atoms or molecules are moving. For example, at the temperature of this room the molecules of air are rushing about with an average speed of 500 yards a second.
>
> (A. S. Eddington, *Stars and Atoms*, O. U. Press, 1927, p. 14.)
>
> Temperatures are expressed throughout in degrees Centigrade. (*Ibid.* p. 6.)

But if temperature indicates how fast atoms and molecules are moving, its expression should make that clear even to those who are not of the physical priesthood.

Transmission in straight lines

The law of rectilinear propagation to which energy in all the forms of wave-motion is subject may be demonstrated by an impressive experiment described by Poynting and Thomson[1]. A tank containing hot water and a thermopile, an instrument which is sensitive to the radiation emitted by hot bodies, are separated by a screen with a hole in it. However much the tank may be moved about backwards and forwards or turned round, the thermopile enjoys exactly the same amount of radiative energy provided that its "eye," the hole in the screen, is always covered by the tank. It can give no indication of anything which is happening on the other side of the aperture. Nothing counts except the temperature and the nature of the surface which fills the aperture as viewed from the thermopile.

The general physical laws of radiation

In considering radiation we are concerned with emission and absorption. There is one important law of radiation, called by the name of Kirchhoff its discoverer, based upon the recognition of simple facts that bodies are always radiating according to their temperature and that within an enclosure maintained at a uniform temperature any substance whatever, whether it be black, or polished, or transparent, will also keep its temperature uniform in consequence of the exchange of radiation between itself and its enclosure. It must therefore be giving out heat by radiation of every kind at precisely the same rate as it is receiving radiation of the same kind from the enclosure. The expression of these conclusions is known as the theory of exchanges of radiation, and is associated with the names of Prévost and Balfour Stewart.

From them it follows that in every particular a good radiator is also a good

[1] *A Text-book of Physics, Heat,* Charles Griffin and Co. Ltd., seventh edition, London, 1922, p. 223.

absorber; a perfect absorber like lamp-black is equally a perfect radiator; contrariwise a bad absorber is also a bad radiator. Hence it is safe to infer that a polished surface of silver, which is a very good reflector, is a very bad radiator compared with lamp-black at the same temperature. A body like rock-salt which is very transparent and therefore a bad absorber is in like manner a bad radiator compared with lamp-black, and as its surface also reflects it is specially bad as a radiator. In respect of the transmission of radiation rock-salt approximates to the idea of a portion of free space.

It must be remembered that radiation and absorption are selective; a substance may be a good radiator for one wave-length and a good reflector for another. Snow, for example, is a nearly perfect reflector of the short waves of light but a nearly perfect radiator of the long waves of heat.

Throughout the whole of the experience of radiation there is the same compensation. All the considerations that are necessary for questions of absorption, absorbing power, reflecting power, scattering power and so on, in relation to wave-length have their counterpart in radiating power at the same temperature. Hence the extraordinary complications of the general question of the balance of loss and gain of heat by radiation in the case of the atmosphere.

We can begin however by considering the phenomena which are related to a black surface, a full absorber or full radiator, in practice a surface which, being coated with lamp-black or some other substance, absorbs and converts into heat, radiation of any length that falls upon it; and in like manner an ideal black surface radiates according to its temperature without regard to other considerations in the manner expressed by Stefan's law, $N = \sigma T^4$, where N is the rate at which total energy is radiated, T the absolute temperature, and σ a constant[1] for which we have the value $5{\cdot}72 \times 10^{-9}$ if we wish the radiation to be expressed in kilowatts per square dekametre.

Stefan's law was originally introduced as a means of expressing observations of the rate of cooling of a body in an environment colder than itself, which had been the subject of experiments by Newton himself and more elaborately by Dulong and Petit. It was deduced by Boltzmann as a necessary consequence by thermodynamical reasoning.

On the theoretical side within the past twenty years the subject of radiation has been developed into a vast literature on the basis of Planck's idea that the energy of radiation is emitted by a radiating body discontinuously, as a succession of very small quanta of action or angular momentum, each $6{\cdot}548 \times 10^{-27}$ erg-sec (O. W. Richardson). The continuous excitation of wave-motion in the aether can no longer be regarded as an adequate representation of the ultimate process of radiation.

The newer views hardly come into consideration in meteorology so long as our attention is concentrated on the thermal effect. We are however concerned with one aspect of the theory in considering the radiation from a

[1] The value of σ is quoted from *Meteorological Glossary*, M.O. 225 ii, 1918, p. 330; the equivalent in g cal/cm^2 min is $8{\cdot}21 \times 10^{-11}$; the *Smithsonian Physical Tables*, 7th revised edition, 1920, p. 247, give $\sigma = 8{\cdot}26 \times 10^{-11}$ g cal/cm^2 min; $5{\cdot}75 \times 10^{-9}$ kw/(10 m)^2.

surface at ordinary temperatures, the radiation which is represented by what we have called long waves. For that we require the use of Wien's law that the wave-length of maximum radiation λ_{max} is inversely proportional to the temperature T or approximately tt of the radiating body,

$$\lambda_{max}\,\text{tt} = 2940 \text{ when the wave-length is measured in micron.}$$

Estimated on this principle the temperature of the surface of the sun is given as about 6000 tt. A table of equivalents of black body radiation at terrestrial temperatures, and the vertical component of solar radiation at different angles of incidence on a horizontal surface, is given in vol. II, p. 1.

Accounting for the general application of Stefan's law to black bodies at meteorological temperatures Planck gave a formula for the distribution of energy between different wave-lengths which may be written[1]

$$\mathcal{J}_\lambda = \frac{C_1}{\lambda^5}\Big/\left(e^{\frac{C_2}{\lambda\text{tt}}} - 1\right),$$

where $\mathcal{J}_\lambda$ is the intensity of radiation for the wave-length λ. When $\mathcal{J}_\lambda$ is expressed in watts per cm^2 per micron, and λ is in microns, $C_1 = 3{\cdot}86 \times 10^4$, $C_2 = 14350$.

For an account of the development of Planck's theory the reader may be referred to Jeans's *Report on radiation and the quantum theory*, Physical Society of London, 1914.

THE SUPPLY OF ENERGY FROM THE SUN

We may now turn our attention to the experience that has been accumulated with reference to the various items of the balance-sheet.

The information which is available about solar and terrestrial radiation is not sufficient for us to generalise the subject in the manner which has become customary with temperature, rainfall or other elements as set out in vol. II. All that we can do is to give an account of the nature and extent of the information which is available and leave the reader to make such generalisations as he finds possible and necessary. The information which is here referred to is based primarily on the representation of the present state of our knowledge of solar and terrestrial radiation which was asked for by the Meteorological Section of the Union for Geodesy and Geophysics at Rome in 1922, and was printed in the Procès-Verbaux of the meeting of the Section at Madrid in 1924. Since the publication the information has been amplified and extended by Dr H. H. Kimball in the *Monthly Weather Review*.

Our first consideration is the amount of energy supplied daily by the solar furnace, called A in the balance-sheet. For the whole earth the mean value is 810 kw-hr/(10 m)^2. For different latitudes and for the middle day of each week of the year the daily total is specified in the table on pp. 4 and 5 of vol. II, which may be briefly recapitulated here:

[1] *Smithsonian Physical Tables*, 1920, p. 247.

Solar radiation on a square dekametre of horizontal surface at the confines of the atmosphere on the middle day of the weeks of equinox and solstice

Latitude	♑ week Dec. 20 kw-hr	♈ week Mar. 22 kw-hr	♋ week June 21 kw-hr	♎ week Sept. 20 kw-hr
90° N	0	30	1249	68
60° „	58	532	1135	541
30° „	540	906	1130	903
0°	977	1038	915	1023
30° S	1207	891	506	869
60° „	1211	506	54	482
90° „	1331	0	0	0

The total radiation which is directed towards a horizontal area of one square dekametre in the course of a year is set out in thousands of kilowatt-hours in the following table:

Total radiation in the year

Latitude	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
	Thousands of kilowatt-hours per square dekametre									
N or S	361	356	341	317	285	247	205	171	155	150

If the energy were devoted to the evaporation of water which would subsequently be condensed elsewhere and expressed as rainfall, the corresponding rainfall would be:

● in cm	520	513	491	456	410	356	295	246	223	216

The fraction which reaches the earth's surface

Our next consideration, for item B of the balance-sheet, is the amount of direct radiation which reaches the earth's surface as derived from the energy absorbed by a pyrheliometer (chap. XII of vol. I) exposed directly to the sun's rays. The instruments of that type in practical use are the Ångström pyrheliometer, the Michelson actinometer, the Marvin pyrheliometer[1], the Abbot silver-disk pyrheliometer, all of which require an observer, and the Gorczynski pyrheliometer which is self-recording.

Standard scale for a "black body"

The object of measurement with all these instruments is the same, namely the amount of energy conveyed in a sunbeam to a "perfectly black" body at the earth's surface. It is usual to think of the black body as a square centimetre of perfectly "blackened" surface, lamp-black, camphor-black, or platinum-black, placed at right angles to the sunbeam.

That ideal is never exactly realised. No kind of smoke, lamp-black or varnish makes a surface which will absorb the whole of every kind of radiation, and neither reflect nor scatter any. The ideal is accordingly secured by the leaders of the Astrophysical Observatory of the Smithsonian Institution in the form of an opening into an enclosure which is blackened inside. Any part

[1] *Monthly Weath. Rev. Washington*, vol. XLVII, 1919, p. 769.

of the radiation that gets into the enclosure through the opening has no more than an infinitesimal chance of getting out again by repeated reflexion, even if part of it is diffused over the interior by reflexion in the first instance.

The readings of the several instruments, if they are to be properly comparable, must be referred to some final standard for a perfectly black body, and for that purpose a scale of comparison for the various instruments has been established by the Smithsonian Institution according to which the reading of an Ångström pyrheliometer has to be multiplied by 1·0325 to bring the results into comparison with the *standard black body of the standard Smithsonian instruments*.

The Michelson actinometer, which is in effect a bimetallic thermometer, requires empirical graduation in any case and introduces no fresh scale of its own. The Gorczynski self-recording pyrheliometer, in which the radiation is received upon a thermopile, is calibrated at the Geophysical Institute at Parc St Maur, Paris. Its readings can be expressed either in the scale of the pyrheliometer of Ångström or in the standard scale of Washington, whichever is regarded as the standard of reference. Either is accepted by the International Meteorological Organisation.

La Commission internationale de la radiation solaire regarde comme de la plus grande importance que le pyrhéliomètre Ångström qui a été accepté comme instrument étalon au Congrès d'Innsbruck, en 1905, soit comparé avec un instrument absolu, construit d'après un principe indépendant. Elle prie donc l'Institut de Météorologie de Potsdam (en collaboration éventuelle avec la Physikalisch-technische Reichsanstalt, Charlottenburg) de diriger son attention sur cette question. C'est aussi très important d'envisager le problème de la construction d'un instrument étalon absolu, destiné uniquement à des étalonnements; et la Commission espère que l'Institut voudra aussi prendre cette question en considération.

(*Rapport de la Réunion de la Commission Internationale de Radiation Solaire tenue à Davos, 1925.* Zürich, 1927, Résolution IV.)

The solar constant

It is upon these measurements that we depend for the values of the solar constant. It is determined by making a correction for the amount of energy intercepted by the atmosphere and thence are derived the mean values upon which the table given above is computed. That is a special department of the study of radiation more definitely related to the study of the sun than that of the atmosphere. Therefore we will not enter into details except to say that the deviation of the amount of solar energy recorded at the Observatory on Mount Wilson from the full amount estimated as incident outside the atmosphere is closely related to the amount of water-vapour in the atmosphere at the time of observation.

We are primarily concerned with the effect of the sun's rays directly or indirectly upon the air, and it will be sufficient for our purpose if we take the solar constant as 135 kilowatts per square dekametre, and refer to chap. I of vol. II for the variations in the constant which have been detected by the specially careful observations of Washington, Mount Wilson and elsewhere.

Apart from any change in the sun there will be a variation in the amount recorded on the earth from about 130·5 to 139·5, on account of the variation in the radius of the earth's orbit in the course of the year.

The local intensity of sunbeams

Item B of the balance-sheet

Observations with one or other of the recognised instruments have been carried out at 99 stations enumerated by H. H. Kimball[1]. Clearly one of the most important questions about any station is the degree to which the energy which is received in a sunbeam approximates to that which is estimated to be incident upon the exterior of the atmosphere; and in answer to that question we have sought the highest values of solar radiation at the various stations.

Variable causes as water-vapour, cloud or dust in the atmosphere may combine at the time of any observation to reduce the amount of solar radiation below the maximum recorded. The highest value thus becomes a "record" of the possibility of the atmosphere at a station, not of its normal character.

Until further information is obtained it may be understood for any purposes of meteorological calculus that the incoming radiation lies between zero and the maximum value which has already been recorded. The outgoing radiation is another story.

In some cases it has not been found possible to obtain from the published data an answer to the simple question which is here asked, although it is of fundamental importance from the meteorological point of view; the heat which any locality receives is beyond dispute a controlling influence in its weather. But many observers with pyrheliometers set themselves to deal with the problem of atmospheric absorption as the primary consideration and take less heed of the energy which is actually placed at the disposal of the locality. Some correct their measures for the altitude of the sun and express the result according to the argument air-mass 1, which we understand to imply the sun in the zenith whether that position happens to be possible or not. Others more realistic prefer air-mass 2. Some correct their observations to the sun at mean distance, others do not; some make allowance in order to refer their observations to the Smithsonian scale and others quote the results obtained by the instrument without that allowance. Some moreover publish mean values only.

In order to represent the results in their geographical environment we sought to show the positions of the several stations on maps of the northern and southern hemispheres with the highest values of the intensity of solar radiation recorded against them; but owing to the irregular distribution of the stations the project of a map has proved impracticable. Instead of it we have formed a table arranged according to zones of latitude 60° to 90°, 45° to 60°, 30° to 45°, 0° to 30°. In the zones the stations are arranged in order of longitude and the maxima recorded are entered in kilowatts per square dekametre.

[1] *Monthly Weath. Rev. Washington*, vol. LV, 1927, pp. 159–60.

Highest measures of direct solar radiation in kilowatts per square dekametre obtained from the results of observations in all parts of the world with pyrheliometers of various kinds. The stations are grouped in zones of latitude, North and South, 60° to 90°, 45° to 60°, 30° to 45° and 0° to 30°. Within the zones they are arranged in order of longitude. References to the original sources of the information are added.

The prefix a means that the values are reduced to mean solar distance and vertical sun, b extrapolated to air-mass 1, c reduced to air-mass 1·2, d reduced to air-mass 1·5, e reduced to air-mass 2, f corrected to local noon, g reduced to mean solar distance, h maximum of mean values, k mean values extrapolated to air-mass 1.

Maximum value	Station	Lat.	Long.	Level in gdm.	Period	Ref. No.
60° to 90° N						
90	Treurenberg	80° N	17° E	9	1899, ix; 1900, iv–vii	1
84	Abisko	68° N	19° E	382	1913, vii–ix	2*a*
92	,,	68° N	19° E	382	1914, vii–viii	2*b*
45° to 60° N						
101	Eskdalemuir	55° N	3° W	232	1911–23	3
99	Richmond (K.O.)	51° N	0°	10	1911–23	3, 4
100	Paris (Parc St Maur)	49° N	2° E	49	1907–23	5
[141]	Mt Blanc	46° N	7° E	4714	1900, vii, ix	6
120	,,	46° N	7° E	4714	1904, viii, ix	6
h 63	Lausanne	47° N	7° E	505	1896–1902	7
114	Jungfraujoch	47° N	8° E	3388	1923, ix, 23–x, 3	8
104	Feldberg	48° N	8° E	1275	1921–5	9
101	St Blasien	48° N	8° E	774	1919–24	9
95	Karlsruhe	49° N	8° E	125	1921–5	9
bh 107	Frankfurt	50° N	9° E	804	1919–22	10
116	Free balloon: Griesheim am Main	51° N	9° E	7350	1913, viii–x (3 days)	11
a 122	,,	51° N	9° E	7350	,,	12
103	Agra (Switzerland)	46° N	9° E	539	1922, x–1923, ix	13
92	Hald	56° N	9° E	76	1902–3	1
?115	Arosa	47° N	10° E	1824	1921–5	14
111	Davos	47° N	10° E	1570	From 1908	15
h 101	Algäu	47° N	10° E	1128	1922, v–1924, v	16
95	Innsbruck	47° N	11° E	568	1908, i–vi	17
115	Brandenburger Haus	47° N	11° E	3211	1928, vi	18
h 112	Sonnblick	47° N	13° E	3045	1902, vi, 19–vii, 17	19
101	Potsdam	52° N	13° E	104	1907–20	20
h 97	Wahnsdorf	51° N	14° E	255	1917, viii–1918, viii	21
87	Lindenberg	52° N	14° E	104	1913, viii–xii & 1919	22
98	Kolberg	54° N	16° E	2	1914, iv–1915, iv	23
97	Nyköping	59° N	17° E	18	1918, iii–1919, v	24
96	Upsala	60° N	18° E	39	1909–13	2*a*, 25
87	Zakopane	49° N	20° E	881	1903, viii–ix	7
102	,,	49° N	20° E	816	1924, i, iv, ix (8 days)	26
e 84	Ursynów	52° N	21° E	98	1909, vi–viii	27
102	Warsaw	52° N	21° E	127	1898–1925	28
91	Worochta	48° N	25° E	755	1924, viii, 19	29
112	Giewont	Tatra		1863	1926, viii, 31	26
114	Świnica	Tatra		2261	1926, ix, 1	26
101	Jabłonica	48° N	24° E	823	1924, viii, 22	29
96	Połonina Pożyżewska	48° N	25° E	1346	1909, ix–xi	29
106	,,	48° N	25° E	1379	1924, vii–viii	29
100	Chomiak	48° N	24° E	1515	1924, viii, 21	29
104	Pożyżewska	48° N	25° E	1787	1924, viii, 17	29
110	Howerla	48° N	25° E	2018	1924, viii, 8 & 31	29
97	Zaleszczyki	49° N	26° E	186	1926, viii, 28–ix, 15	26
105	Nijni-Oltchedaeff	49° N	28° E	193	1912–15	30
102	Leningrad	60° N	30° E	5	1895–1904	1
97	Kief	50° N	30° E	179	1888	1
103	Pavlovsk	60° N	30° E	39	1893–1906	1
100	,,	60° N	30° E	39	1906–26	31
f 101	Théodosie	45° N	35° E	—	1926, i–1928, viii	32
103	Moscow	56° N	38° E	153	1914–24	33
110	Katharinenburg	57° N	61° E	284	1896–8	1
30° to 45° N						
b 108	Medford	42° N	123° W	459	1920, iii, 28–iv, 5	34
b 105	Red Bluff	40° N	122° W	108	1920, iii, 23–25	34
b 106	Fresno	37° N	120° W	108	1920, iii, 14	34
a 120	Mt Whitney	37° N	118° W	4329	1908, viii; 1909, ix; 1910, viii	12
a 114	Mt Wilson	34° N	118° W	1691	1905–20	12
b 111	Pomona	34° N	118° W	260	1920, ii, 26–28	34
b 109	La Jolla	33° N	117° W	29	1920, iii, 2–4	34
103	Phoenix	33° N	112° W	329	1910, x, 2–8	35
109	Flagstaff	35° N	112° W	2061	1910, ix, 25–30	35
b 111	,,	35° N	112° W	2061	,,	35
116	Santa Fe	36° N	106° W	2094	To 1919	35
b 123	,,	36° N	106° W	2094	1912–22	36, 37
104	Twin Mt.	36° N	106° W	2388	1912, x, 25	38
99	Lake Peak	36° N	106° W	3643	1912, x, 29	38
102	Cheyenne	41° N	105° W	2063	1910, viii, 29–ix, 3	35
b 121	Lincoln	41° N	97° W	366	1910–28	37, 39
110	,,	41° N	97° W	366	To 1919	35
a 128	Free balloon: Omaha	41° N	96° W	21500	1914, vii, 11	12

Maximum value	Station	Lat.	Long.	Level in gdm.	Period	Ref. No.
b 118	Madison	43° N	89° W	291	1910–28	36, 37
112	,,	43° N	89° W	291	To 1919	35
b 100	Ellijay	35° N	83° W	669	1916, v, 8–13	35
110	Hump Mt.	36° N	82° W	1470	1917, vi–1918, iii	40
102	Toronto	44° N	79° W	114	1910–24	41
b 107	Mt Weather	39° N	78° W	529	1907–14	36, 42
105	,,	39° N	78° W	529	1907–14	35
d 92	Trapp	39° N	78° W	216	1909, viii, 30–ix, 2	38
b 118	Washington	39° N	77° W	124	1914–28	37
105	,,	39° N	77° W	124	To 1919	35
97	Atlantic Ocean	38° N	10° W	—	1923, iii, 8	43
102	Madrid	40° N	4° W	642	1910–20	44
103	Bassour	36° N	3° E	1137	1911, viii–xi	45
111	Ouargla	32° N	3° E	ca. 0	1926, i–iv	46
112	Montpellier	44° N	4° E	42	1883–1900	1
105	,,	44° N	4° E	42	1924–7	47
101	Tougourt	33° N	6° E	?	1924, iii, 23–iv, 15	48
112	Ariana	37° N	8° E	10	1924–7	47
96	Modena	45° N	11° E	50	1900–3	1
92	Florence	44° N	11° E	72	1915, vi–1917, xii	49
95	Naples	41° N	14° E	146	1913, xii–1915, i	50
102	Etna	38° N	15° E	2890	1908, viii, 22–23	51
88	,,	38° N	15° E	1846	1908, viii, 21–23	51
100	,,	38° N	15° E	1846	1908, viii, 18–19	51
86	,,	38° N	15° E	739	,,	51
g 112	Mt Elbrus	43° N	42° E	3135	1926, viii, 8–20	52
96	Mediterranean	—	—	—	1923, iii, 13 & viii, 5, 7, 9	43
f 103	Tashkent	41° N	69° E	?	1926, i–1928, viii	32
f 112	Simla	31° N	77° E	2158	1906–16	53
109	Fujiyama	35° N	139° E	3648	1909, vii, 29	54
90	Numazu	35° N	139° E	10	,,	54
0° to 30° N						
116	Tacubaya	20° N	99° W	2259	1911–15	47
	N. Atlantic:				1923, iv, vii, viii	55
k 101	NE trades	—	—	—	—	

Maximum value	Station	Lat.	Long.	Level in gdm.	Period	Ref. No.
k 97	N. temp. zone	—	—	—	—	
k 93	Calm zone	—	—	—	—	
k 78	Off C. Verde Is.	—	—	—	—	
	Tenerife:					
114	Alta Vista	28° N	17° W	3183	1896, vi, 21–vii, 3	7
116	Izana	28° N	17° W	2317	1916, iv–xii	56
119	Cañadas	28° N	17° W	2055	1912, v–1915, vi	57
99	Guimar	28° N	16° W	353	1896, vii, 2–3	7
85	Suez Canal	29° N	33° E	—	1923, iii, 18	43
86	Red Sea	—	—	—	1923, iii, 20; vii, 31; viii, 1	43
95	Gulf of Aden	—	—	—	1923, iii, 23; vii, 28	43
95	Indian Ocean	—	—	—	1923, iii, 28; vii, 22	43
87	Bangkok	14° N	101° E	10	1923, v (4 days)	43
89	Gulf of Siam	3° N	101° E	—	1923, iv, 10	43
0° to 30° S						
h 89	Apia	14° S	172° W	2	1925–7	58
gh 86	,,	14° S	172° W	2	,,	58
118	Calama	22° S	69° W	2202	1918, vi–1920, vii	40
c 113	Arequipa	16° S	72° W	2397	1912, viii–1915, iii	59
k 111	Argentina	—	—	?	1923, v–vii	55
k 111	Andes	—	—	2640	,,	55
k 113	Bolivian plateau	—	—	3520	,,	55
	S. Atlantic:				1923, iv, vii, viii	55
k 101	S. temp. zone	—	—	—	,,	
k 102	SE trades	—	—	—	,,	
b 125	La Quiaca	22° S	66° W	3390	1912, ix–1913, x	60
112	Johannesburg	26° S	28° E	1767	1907, iv–1910, vi	61
h 98	Batavia	6° S	107° E	8	1915; 1917–19	62
112	Pangerango	7° S	107° E	2956	1923, vi, 15–17	43
114	,,	7° S	107° E	2962	1915 & 1919 (4 days)	62
99	Tjisoeroepan	7° S	109° E	1174	1918, vii (7 days)	62
117	Smeroe	8° S	113° E	3588	1915, iv, 30; 1918, viii, 25	62
30° to 60° S						
103	Cape Horn	56° S	70° W	12	1882, ix–1883, ix	1
b 110	Cordoba	31° S	64° W	429	1912, ii–1914, vi	60

Supplementary values

60° to 90° N						
97	Jungfruskär	60° N	21° E	ca. 10	1922–6	65
98	Wirrat	62° N	24° E	ca. 88	,,	65
97	Antrea	61° N	29° E	ca. 15	,,	65
0° to 30° N						
b 83	Caribbean Sea	16° N	85° W	—	1925, vii, 30	64
b 83	N. Atlantic	21° N	53° W	—	1925, viii, 16	64
b 92	N. Atlantic	27° N	44° W	—	1925, v, 30	64
86	NE trades	17–18° N	24–25° W		1924, vi, 6	63

0° to 30° S					
89	SE trades	11–23° S	36–41° W	1924, iv, 4, 7	63
30° to 60° S					
90	S. sub-tropic	32–33° S	50–51° W	1924, iv, 29	63

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If we assume the solar constant to be 135 kilowatts per square dekametre we may conclude from the information displayed in the table that an intensity of 100 kw/(10 m)², three-quarters of the intensity at the exterior of the atmosphere, represents a high standard for "strong sunshine." No station in the whole world records as energy received at the surface radiation equal to the solar constant; lack of altitude, latitude and other local conditions of climate interfere to reduce by scattering or by absorption the amount of direct radiation received.

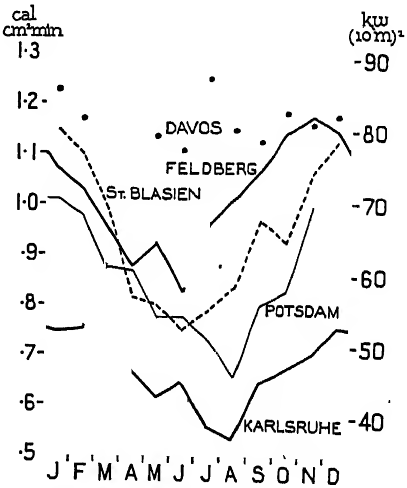


Fig. 51. Seasonal variation in the normal intensity of radiation with the sun at an altitude of 15° at three stations in Baden at different elevations, viz. Karlsruhe 128 m, St Blasien 790 m, Feldberg 1300 m, with corresponding values at Potsdam 106 m and Davos 1600 m (ten of the months only) for comparison. (Peppler, ref. no. 9, p. 119.)

It will be noticed that the figures arrange themselves primarily according to altitude and latitude. The influence of altitude may be regarded as representing the freedom of the air from dust and water-vapour, and the influence of latitude may perhaps be traced to the inverse cause, but brought to account rather by length of path than by the amount of vapour in a vertical column; the faint nebulosity of the sky in high latitudes may also have an effect. The seasonal variation of intensity of radiation for the same solar altitude at stations at different heights in Baden with Potsdam for comparison and some incomplete monthly figures for Davos is exhibited in a diagram (fig. 51) by the brothers Peppler. The influence of height is obvious but not quite simple; there are clearly some disturbing causes.

A conspicuous common feature of the curves which compose the diagram is the diminution of the intensity in the summer months as compared with winter for the same solar altitude. At Karlsruhe the minimum is in August as at Potsdam; in June at St Blasien and also on the Feldberg. Davos too has a minimum in June but in sharp contrast with it an interpolated figure for July, the highest of the year. The seasonal difference amounts to between 20 and 30 kilowatts per square dekametre or fifty per cent. of the mean value.

It is presumably to be attributed to dust or water-vapour appropriate to the low altitude of 15°.

Seasonal and diurnal variation of the intensity of sunbeams

Further particulars about solar radiation at direct incidence are available; as a rule stations in Europe give the normal value of the intensity of direct radiation on a surface perpendicular to the sun's rays at the hours of the day in each of the twelve months. These may be regarded as the fundamental data for this part of the subject.

In the United States of America, where a good deal of attention has been directed to obtaining a correction of the crude observations in order to get a measure of the solar constant, the length of path of the sunbeam through the earth's atmosphere is regarded as a more effectual datum than the time of day. The actual length of path of a beam is directly related to the sun's altitude. Neglecting the curvature of the earth's surface and regarding the atmosphere as made up of a succession of horizontal layers, the length of path through the atmosphere is the length along the vertical, multiplied by the secant of the angular distance of the sun from the zenith. In this way there is introduced the idea of "air-mass" traversed by the beam. For an inclined beam the air-mass is proportional to the length of path and is related to that for a vertical beam by what is known as the "secant law." The expression in terms of air-mass can be rendered more precise by taking account of the earth's curvature, refraction, etc., and hence we obtain the following:

Relation of air-mass and solar altitude

Besides values derived from the pure secant formula, the table contains those derived from various other more complex formulae, taking into account the curvature of the earth, refraction, etc. The most recent is that of Bemporad.

Solar altitude	90°	70°	50°	30°	20°	15°	10°	5°	2°
					Air-mass				
Secant law	1·00	1·064	1·305	2·000	2·924	3·864	5·76	11·47	28·7
Forbes	1·00	1·065	1·306	1·995	2·902	3·809	5·57	10·22	18·9
Bouguer	1·00	1·064	1·305	1·990	2·900	3·805	5·56	10·20	19·0
Laplace	1·00	—	—	1·993	2·899	—	5·56	10·20	18·8
Bemporad	1·00	—	—	1·995	2·904	—	5·60	10·39	19·8

(From *Smithsonian Physical Tables*, 7th revised edition, 1920, p. 419)

As the "unit air-mass" is that of a vertical column at the station, and the sun is never vertically above a point north of the tropic of Cancer (23½° N), or south of the tropic of Capricorn, the value of radiation for unit air-mass at a station outside the tropics requires extrapolation.

Here the reader may require a table showing the range of variation of the values of the air-mass at midday in different latitudes.

Maximum and minimum values at midday in various latitudes of the air-mass, according to the secant of the zenith distance

Latitude ...		0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Air-mass:	Min.	1·000	1·000	1·000	1·007	1·043	1·118	1·245	1·454	1·814	2·513
	Max.	1·090	1·199	1·377	1·679	2·237	3·511	8·77	—	—	—

In order to illustrate the difference between the two modes of treatment of the data we have endeavoured to form a table of intensity of radiation at fixed hours from the information contained in one of the tables of solar radiation in relation to air-mass, namely that of Washington.

With this object we have computed the time of day in the several months at which the sun's altitude would correspond with an air-mass of 2, 3, 4, 5 respectively and have prepared a table showing the figures of radiation for these air-masses at Washington arranged according to a time-scale similar to that of European stations. The result is given in fig. 52. It will be noticed that with the arrangement according to air-mass, for stations where there is no air-mass 1, quite a number of hours in the middle of the day in the summer months have no information as to solar radiation, the first line of figures corresponds with air-mass 2, and all the information being concentrated within the time of air-mass 2 and air-mass 5, belongs to a very short period in the morning or the afternoon. We have made an attempt to supply the missing information by quoting from another source the noon values of radiation.

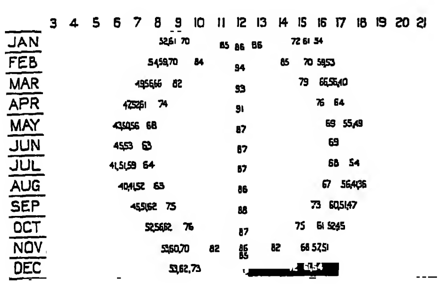


Fig. 52. Solar intensity, air-mass and time of day. Washington.

The figures have to be so compact that they are difficult to read, the same figures appear in the adjacent table.

Intensity of solar radiation in kilowatts per square dekametre at different solar altitudes arranged according to air-mass

Santa Fe. 36° N, 106° W, 2145 m. 1912–1920

air-mass	5	4	3	2	1	2	3	4	5
Jan.	—	89	96	105	—	102	93	84	80
Feb.	—	85	91	102	—	98	91	82	75
Mar.	—	84	92	101	—	97	87	77	72
Apr.	63	75	82	93	107	86	75	71	61
May	70	74	82	90	106	86	93	—	—
June	—	66	74	86	102	92	81	70	—
July	—	66	75	84	99	—	—	—	—
Aug.	64	68	77	86	100	89	82	76	—
Sept.	67	74	81	91	107	97	86	79	70
Oct.	75	80	86	96	—	98	86	77	61
Nov.	81	84	95	102	—	102	91	83	—
Dec.	79	87	95	105	—	103	91	84	75
Solar alt	11°	14	19	30	90	30	19	14	11

Lincoln. 41° N, 97° W, 373 m. 1915–1920

air-mass	5	4	3	2	1	2	3	4	5
Jan.	66	73	82	95	—	87	84	75	70
Feb.	72	76	86	99	—	94	83	72	62
Mar.	64	64	76	91	—	89	76	66	56
Apr.	54	58	70	86	104	81	68	58	49
May	—	57	67	79	96	75	62	52	49
June	—	53	64	75	95	77	63	54	—
July	—	56	63	76	93	75	62	52	—
Aug.	49	56	62	74	93	77	63	54	49
Sept.	53	59	69	83	97	80	68	59	52
Oct.	63	68	78	90	—	87	75	66	59
Nov.	68	75	85	95	—	97	84	74	66
Dec.	63	73	85	97	—	—	84	75	68
Solar alt	11°	14	19	30	90	30	19	14	11

Madison. 43° N, 89° W, 297 m. 1910–1920

air-mass	5	4	3	2	1	2	3	4	5
Jan.	66	75	87	94	—	—	86	79	—
Feb.	—	—	86	96	—	98	84	—	—
Mar.	—	74	84	93	—	93	82	75	—
Apr.	—	—	—	86	98	87	75	61	47
May	—	—	63	77	93	72	63	—	—
June	—	61	67	79	91	74	61	—	—
July	44	53	61	70	87	68	63	—	—
Aug.	49	58	64	76	91	72	60	56	43
Sept.	63	63	70	82	95	81	70	59	—
Oct.	49	62	73	81	—	82	70	62	—
Nov.	61	70	80	91	—	94	82	—	—
Dec.	65	79	85	—	—	—	—	75	—
Solar alt	11°	14	19	30	90	30	19	14	11

Washington. 39° N, 77° W, 127 m. 1905–1920

air-mass	5	4	3	2	1	2	3	4	5
Jan.	52	61	70	85	—	86	72	61	54
Feb.	54	59	70	84	—	85	70	59	53
Mar.	49	56	66	82	—	79	66	56	40
Apr.	47	52	61	74	96	76	64	—	—
May	43	50	56	68	90	69	55	49	—
June	—	45	53	63	89	69	—	—	—
July	41	51	59	64	86	68	54	—	—
Aug.	40	41	52	62	86	67	56	41	36
Sept.	45	51	62	75	92	73	60	51	47
Oct.	52	56	62	76	—	75	61	52	45
Nov.	53	60	70	82	—	82	68	57	51
Dec.	53	62	73	85	—	—	72	61	54
Solar alt	11°	14	19	30	90	30	19	14	11

Observations with a Marvin pyrheliometer. The values for air-mass 1 are extrapolated.

Fig. 53. Davos. 47° N, 10° E, 1600 m. 1912–1918

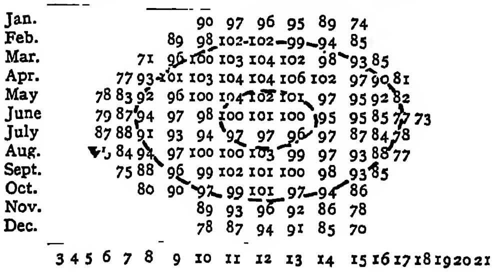


Fig. 54. Taunus. 50° N, 9° E, 820 m. July 1919 to Mar. 1922

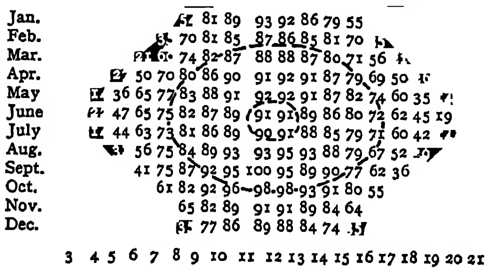


Fig. 55. Potsdam. 52° N, 13° E, 106 m. 1907–1923

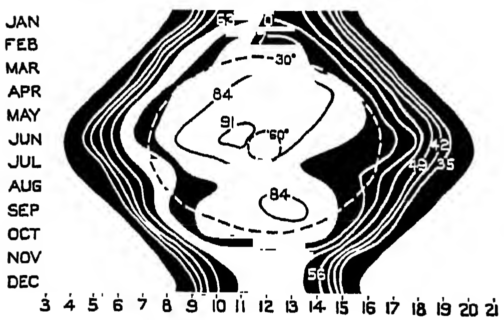
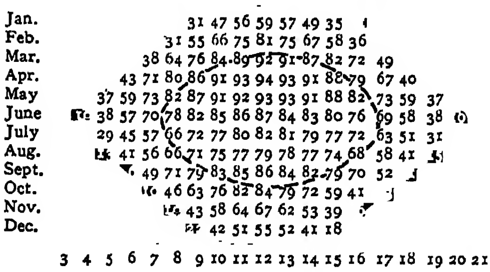


Fig. 56. Kolberg. 54° N, 16° E, 2 m. Ap. 1914 to Ap. 1915



Figs. 53–56. Diurnal and seasonal variation of the intensity of solar radiation in kw/(10 m)^2. The broken lines indicate the points on the diagram when the altitude of the sun is 30° and when 60°.

The atmosphere is not generally amenable to an algebraical formula and therefore we should ourselves prefer the measures of solar radiation arranged according to the hours of the day. However in the absence of data on that basis we give on p. 122 tables in which air-mass is the basis of reference for Madison Wis.; Santa Fe, New Mexico; Lincoln Neb.; and Washington D.C.; and overleaf (p. 124) we give corresponding tables, in which the arrangement is primarily according to solar altitude, for Naples by A. Bemporad in which the data are grouped by quarters instead of months, and for Batavia, Java. Overleaf too we have given a table of solar radiation at Tenerife (Las Cañadas del Teide) for a year which includes the period in which there was obscurity due to Katmai. It shows a depreciation of the solar radiation in August which may have been due to that cause, though the midsummer months on this page are characterised by a fall in the value of the intensity of sunshine.

For illustration of the general result of observations throughout the year we give (figs. 53–56) numerical results which show the diurnal and seasonal variation at four typical stations in Europe, namely: Kolberg, sea-level, temperate zone; Potsdam, low elevation, temperate zone; Taunus (Frankfurt a/Main), middle level, continental, temperate zone; Davos, continental, high level, temperate zone.

Taking the tables and diagrams together there are some points which the reader will have no difficulty in verifying.

First, elevation is a great advantage for productive radiation; the highest of the four European stations, Davos at 1600 m, is the only one at which the normal intensity of radiation reaches 100 kw/(10 m)^2, 75 per cent. of the solar constant. Santa Fe, at the great height of 2145 m, which also surpasses 75 per cent., is the most intensely radiated station of the four in the United States.

Intensity of solar radiation in kilowatts per square dekametre at different solar altitudes

Naples. 41° N, 16° E, 149 m: December 1913–January 1915

Solar altitude a.m.	3°	5	10	20	30	60	60	30	20	10	5	3 p.m.
Winter	17	27	46	66	77	90	87	74	63	43	25	15
Spring	13	22	41	61	72	85	88	74	63	40	20	11
Summer	12	22	41	62	72	83	83	72	62	41	21	10
Autumn	22	34	53	72	82	92	96	79	67	47	29	20

Observations with an Ångström pyrheliometer. The values for solar altitude 60° in winter and autumn are extrapolated.

Java: Batavia. 6° S, 107° E, 8 m: 1915, 1917–19

Solar altitude	20°	25	30	35	40	45	50	55	60	65	70	75	80	85	90°
Jan.	—	73	79	84	88	90	92	93	93	94	95	—	—	—	—
Feb.	—	—	—	82	86	87	89	89	89	90	91	91	92	92	—
Mar.	68	68	72	77	81	83	86	88	92	91	93	94	96	96	95
Apr.	60	73	79	82	84	85	88	90	91	92	93	94	93	95	—
May	—	69	68	73	77	82	85	86	87	88	88	—	—	—	—
June	58	65	69	72	77	80	82	84	85	87	—	—	—	—	—
July	53	59	65	68	72	76	79	82	83	88	—	—	—	—	—
Aug.	48	53	60	65	69	74	77	79	80	82	81	—	—	—	—
Sept.	37	46	62	66	69	72	74	77	78	79	80	81	84	77	—
Oct.	50	49	55	59	62	65	67	69	70	79	80	81	81	73	70
Nov.	53	68	73	75	79	81	84	88	90	91	97	97	—	—	—
Dec.	—	67	74	80	85	87	90	92	95	96	97	98	—	—	—

Observations with a silver-disk pyrheliometer from Washington and a pyrheliometer constructed on the Michelson principle calibrated by the Washington instrument.

The sun's altitude, or its geometrical equivalent the length of path or air-mass, is not the only factor of consideration. With the same altitude larger measures are got in the spring than in the autumn at Davos; a midsummer falling off of the maximum radiation for the day is manifested. Santa Fe has the same characteristic. Corresponding results occur at the other stations with a smaller average normal intensity of sunshine. Autumn is the favourable season for intensity of radiation at Naples, and in Java the intensity becomes quite marked in November after a decrease accumulated during the dry season, June, July, August and September. Robitzsch[1] has pointed out that for the same altitude of the sun the results at Lindenberg show higher values of intensity in the afternoon than in the morning in winter (January), while in the summer the highest intensities are in the morning.

Diurnal and seasonal table of the intensity of solar radiation in kilowatts per square dekametre

Las Cañadas del Teide. 28° N, 17° W, 2100 m

h	7	8	9	10	11	12	13	14	15	16	17	18
1912 June	—	—	97	105	104	105	106	104	101	96	73	61
July	—	96	99	104	107	106	105	106	102	96	85	67
Aug.	*63*	83	92	98	98	98	97	96	90	82	67	52
Sept.	—	80	92	98	99	100	98	96	92	79	66	—
Oct.	—	*80*	95	100	102	102	101	97	90	76	—	—
Nov.	—	—	92	99	103	101	101	98	88	77	—	—
Dec.	—	—	93	96	102	104	103	98	88	*85*	—	—
1913 Jan.	—	—	90	95	102	102	102	98	90	*73*	—	—
Feb.	—	—	95	97	101	104	103	100	95	84	—	—
Mar.	—	*81*	*91*	101	103	104	103	99	93	82	—	—
Apr.	86	95	100	105	107	108	107	105	102	94	80	—
May	87	94	101	104	106	105	104	102	98	91	79	79
June	86	93	99	102	104	104	103	102	98	91	79	58

Values in italics are based on less than 10 observations

[1] 'Einige Ergebnisse von Strahlungsregistrierungen, die im Jahre 1919 in Lindenberg gewonnen wurden,' *Beiträge zur Physik der freien Atmosphäre*, Bd. IX, 1920, p. 91.

The differences are explained by the absorption of solar radiation by certain constituents of the atmosphere of which water-vapour and dust-haze are the chief. We may illustrate the relation by a diagram (fig. 57 *a*) taken from Pavlovsk which shows the variation of vapour-pressure during the year 1917, the rainfall and the coefficient of transmission of solar radiation. A corresponding result may be found in the results obtained at the Solar Observatory of the Smithsonian Institution where, with good reason, it is regarded as possible to employ an observation of the solar radiation to give a measure of the humidity of the local atmosphere. But the relation is not general, as the diagram (fig. 57 *b*) of observations at Java of radiation, rainfall and vapour-pressure shows.

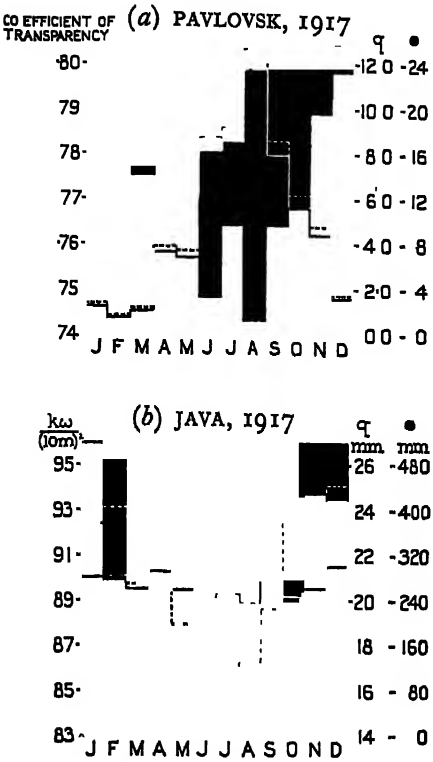


Fig. 57 The reciprocal relation of transparency of the atmosphere at Pavlovsk and Batavia, Java, with vapour-pressure q and rainfall •; monthly values for 1917 are represented.

Transparency at Pavlovsk, or intensity of radiation for solar altitude 60° at Batavia, are shown by the lower limit of the black columns read on a scale on the left. Vapour-pressure q, by the black full line (continued in white over the black ground) according to a scale of millimetres on the right.
Rainfall • by the interrupted line according to another scale of millimetres on the right; that in (*b*) is for Western Java

J. Boerema writes of Java: "As in the wet season the solar radiation at Batavia is stronger than during the dry monsoon it is evident that the effect of the haze is much stronger than that of the vapour-pressure. The rains wash the haze and dust-particles out of the atmosphere."

The idea that rain washes the solid particles out of the atmosphere is reinforced in a paper on 'Blue sky measurements at Washington' by Irving F. Hand, to which reference has been made already. From which it appears that immediately following rain the visibility was highest, so also very notably was the force of the wind, the skylight polarisation and the solar radiation, whereas the vapour-pressure was least as well as the number of dust-particles.

It does not follow necessarily from these conclusions that rain washes out dust-particles. To make that inference clear one would have to follow the dusty air. In the actual observations the air which is observed to be clear is not the same air as that which was observed to be hazy. Quite frequently rain occurs when a supply of a different kind of air is arriving. Nevertheless it is safe to conclude that air which is left in conditions which are not rainy does accumulate a considerable load of dust.

Accidental influences—the dust of volcanoes

Parts of items C and D

For solar radiation as for other elements normal values have not always a definite meaning; the transparency of the atmosphere is subject to considerable fluctuations which can only be called accidental. An example of the "accidental" variation in the annual mean for Pavlovsk[1] is shown in fig. 58, which indicates the peculiar opacity of the sky in 1912.

The remarkable effect was noticed in many parts of the world as bringing a feebleness of sunshine, a paleness of the blue of the sky and a decrease of the intensity of radiation which came on suddenly about the middle of the year and was attributed to the loading of the atmosphere with dust by the

THE INFLUENCE OF DUST ON INTENSITY OF SOLAR RADIATION

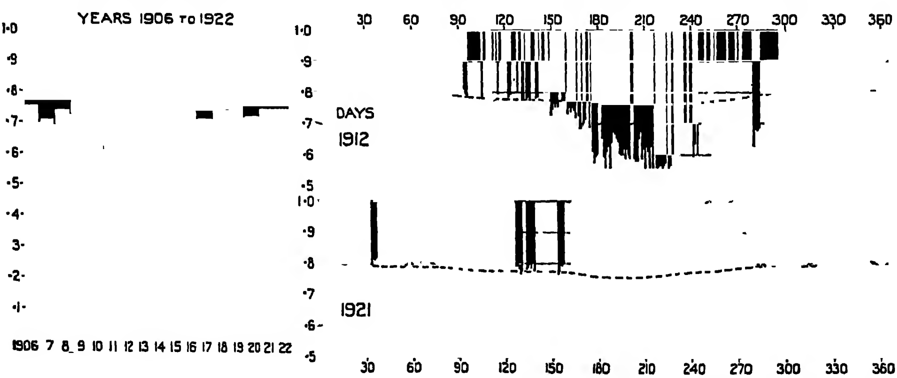


Fig. 58. Diagrams for Pavlovsk showing the effect attributed to the eruption of Katmai, Alaska, which occurred on the 158th day of 1912 (6 June). On the left the mean value of the energy which would be registered for zenith sun, in decimal fractions of the incident solar energy. On the right the observations in 1912 from which the mean is obtained, and beneath them the observations of the very sunny year 1921. The dotted line which is repeated in the diagrams of daily values represents the normal for the years 1912–22.

eruption of Katmai, a volcano in Alaska. It was apparently so clearly traceable to that cause that we may use this opportunity of illustrating the disturbing influence of volcanoes.

Many references to the subject are to be found in meteorological literature. It is treated in a very engaging manner by W. J. Humphreys in his volume on the *Physics of the Air*. The Smithsonian Institution has devoted a volume of its publications to a report on the subject by Abbot and Fowle[2].

In order to show the contrast between two typical years we have amplified fig. 58 by a diagram representing daily observations at Pavlovsk in 1912 the year of the Katmai eruption, and in 1921 the year of brilliant summer in England.

[1] N. N. Kalitine, *Recueil de Géophysique publié par l'Observatoire Géophysique Central*, tome IV, fasc. 3, Leningrad, 1925.

[2] 'Volcanoes and Climate,' *Smithsonian Misc. Coll.* vol. LX, No. 29, 1913, reprinted in *Annals of the Astrophysical Observatory*, vol. III, 1913.

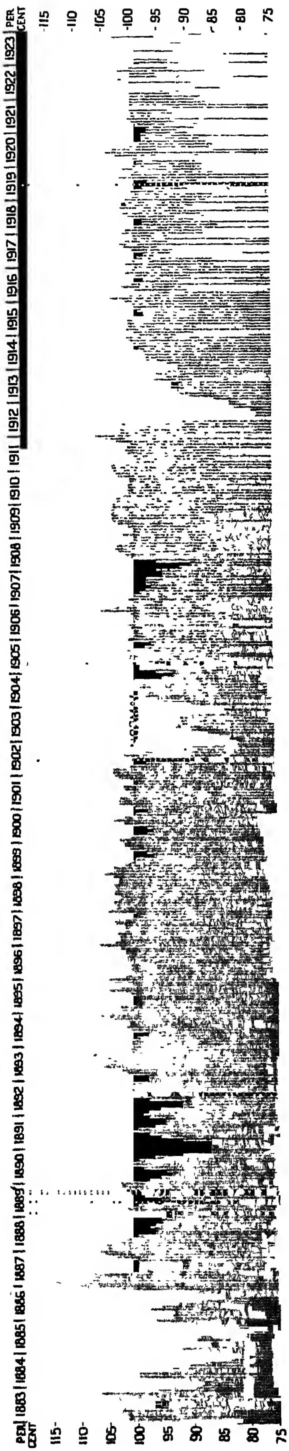


Fig. 59. Diagram of the screening power of the atmosphere in middle latitudes of the northern hemisphere month by month from 1883 to 1923 inclusive, representing data compiled by H. H. Kimball from observations at thirteen stations in the United States, Mexico, Europe, Egypt and India.

The boundary between black and white in the column for each month gives the average solar radiation at the earth's surface in that month expressed as the percentage of the normal and shown on the scales at the side.

The chequered lines indicate the months for which no data are available.

The subject is more fully illustrated by a diagram (fig. 59) based on material compiled by Kimball[1] which, with a few exceptions, shows the monthly values of solar radiation for a period of 41 years, and incidentally illustrates the relation of solar radiation to volcanic eruptions on previous occasions. The occasions of eruptions are indicated in a diagram introduced on p. 278 of vol. I to illustrate curve-parallels. The years of the more important volcanic eruptions since 1800 are given in a table on p. 25 of vol. II. Those which correspond with the three marked periods of atmospheric screening in Kimball's diagram are Krakatoa in August 1883, Pelée, Santa Maria and Colima in 1902 and Katmai in June 1912.

In this connexion we recall some notes made by Kimball[2] in discussing similar data, for Mount Weather:

> There are not many hours at any season in the year, and specially during the summer, when the sky at Mount Weather is free from clouds. It can therefore only be claimed that the [available] data indicate that with a cloudless sky the total radiation received on a horizontal surface during September and October 1912 averaged about 5 per cent. less than during the same months in 1913, and during November and December 1912, about 3 per cent. less than during the corresponding period in 1913....

A similar comparison of the intensities of direct solar radiation gives deficiencies for 1912 twice as great, or 10 per cent. in September and October, and 6 per cent. in November and December.

[1] 'Variation in solar radiation intensities measured at the surface of the earth,' *Monthly Weath. Rev. Washington*, vol. LII, 1924, p. 527.

[2] *Bulletin of the Mount Weather Observatory*, vol. VI, 1914, p. 207.

Attention is called to these remarks because the additional short-wave radiation that reaches a horizontal surface in consequence of the scattering by the dust-particles seems to go some way towards compensating the surface for the loss of direct radiation on account of obstruction by the dust.

Compensation of that kind is noticed by A. Ångström in his work on air-

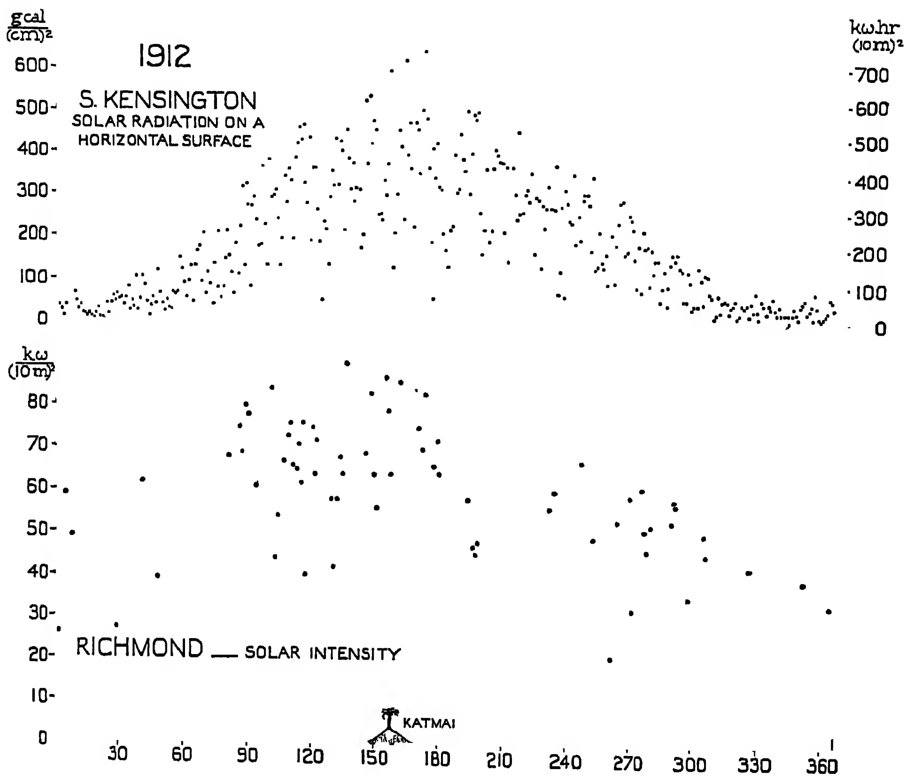


Fig. 60. (1) Daily values of energy received as short-wave radiation on a horizontal surface at South Kensington in 1912 in which the effect of the eruption of Katmai is not easily detected.

(2) Occasional observations of the intensity of direct solar radiation at Kew Observatory, Richmond, in the same year, in which the effect of the eruption on the 158th day is shown partly by the absence of observations (which were only taken on days recognised as being notably sunny) and by the low values obtained when observations were taken between the 181st and the 245th day.

radiation in connexion with the observations at Bassour which were made during 1912, the year of a dusty atmosphere. A similar conclusion may be drawn from plots of the curves of 14-day-values of radiation from sun and sky at Mount Weather in the years 1912 and 1913 which show no conspicuous difference, and from corresponding monthly values for the same years from South Kensington which make the year 1912 to provide the more abundant supply of energy.

The fortnightly and still more obviously the monthly values which are here referred to are obtained from very irregular material. In illustration of this fact in connexion with the compensation by diffuse radiation for the loss of direct radiation, we give a diagram of the daily totals of radiation at South Kensington in 1912, and of the occasional observations of the intensity of solar radiation at Richmond (Kew Observatory) in the same year (fig. 60).

The reader will probably agree that the ranges in this diagram over which means would have to be taken are so wide that the significance of a mean value becomes very dubious, and he will accept the conclusion that the influence of radiation as the fundamental agency in the working of the atmospheric engine is a complex matter many details of which are not disclosed in any general balance-sheet.

It may be thought that in this section an unnecessary amount of attention has been devoted to the single incident of the eruption in Katmai, which counts, after all, as a comparatively small item among the forces which affect weather; but it is upon the consideration of special occurrences of that kind that the selection of a line of approach to the solution of the general meteorological problem must depend. In that case a direct relation could be traced between the solar radiation and weather in many parts of the world which would have been unnoticed in the general scheme of mean values of ordinary meteorological data.

Total radiation upon a horizontal surface

Item B of the balance-sheet

We have allowed the question of compensation by scattering for loss by the obstruction due to dust to forestall to a certain extent the more general question of the total receipt of radiation on a horizontal surface.

It is not merely by the direct sunbeam that the earth is affected. The radiation scattered by the air or by the clouds of water or dust that float in it, or by mountains near the instrument which may be snow-covered, is also effective in communicating the heat from the sun, as furnace, to the earth as the boiler of the atmosphere. The energy which comes in this way as radiation from the sky is usually referred to as sky-radiation and is thereby distinguished from the spontaneous radiation from the air-molecules.

From a combination of the readings of the pyrheliometer and the pyranometer, figures for the ratio of sky-radiation to that of a sunbeam can be derived.

The total radiation from the sun and sky together is shown upon the Callendar sunshine recorder (see vol. I, p. 238). Corresponding results can be obtained by Ångström's pyranometer[1]. In the design of the latter, opportunity has been taken to obviate some of the difficulties in the working of the Callendar instrument, notably the liability to deterioration of the absorptive power of the lamp-black-covering of the one half of the exposed surface. In both instruments a glass cover protects the sensitive parts of the receiving

[1] *Monthly Weath. Rev. Washington*, 1919, p. 795, also *Meddel. från Stat. Meteor.-Hydrog. Anst.* Band 4, No. 3, Stockholm, 1928. Abbot and Aldrich have also a pyranometer, *see* vol. I, chap. XII.

apparatus; and therein is a source of difficulty. It is allowed to absorb any long-wave radiation that falls upon it and is expected to transmit the short waves; but the cover is liable to become dirty and in any case some of the radiation is reflected or absorbed by the glass. The Callendar instrument relies for the accuracy of its measurements upon expert calibration with an Ångström pyrheliometer as standard.

A comparison between this instrument [the Ångström pyranometer] and the pyranometer of Abbot and Aldrich showed that the difference between the readings of the two instruments is less than 2 per cent. Individual readings differ, however, by as much as 6 per cent. due, according to my opinion, to the fact that the pyranometer readings are influenced by the heating of the glass screen.

A comparison with the Callendar recording instrument...showed also a satisfactory agreement in the averages. The Callendar readings were, however, under conditions of very calm weather, undoubtedly influenced by the heating of the glass, the convection of the heat from the glass through the air being then small. The effect is generally not a large one, but may under special conditions amount to as much as 10 per cent.

(A. Ångström, *Monthly Weath. Rev.* vol. XLVII, Washington, 1919, p. 797. For a discussion of 'Some characteristics of the Callendar pyrheliometer' *see* E. R. Miller, *ibid.* vol. XLVIII, 1920, p. 344.)

The effective record of a pyranometer of any pattern that will give accurate results and a comparison with a record of direct radiation are fundamental considerations in regard to the heat received at the earth's surface or, in other words, of the action of the furnace on the boiler. We have already displayed the results of observations of direct radiation in different parts of the world and we may now consider the results obtained from records with the pyranometer.

From the combination we can get the amount which might be received upon a horizontal surface from the sun and sky together, as well as the amount derived from the sun alone. From the differences between these two we can obtain as a separate item the amount received in the form of short-wave radiation from the sky.

For Stockholm Ångström has put together the double information. The result is represented in fig. 61, following a plan which is also applicable when the direct radiation is unknown.

The scheme of representation is first to show the primary supply of solar radiation in the course of the year outside the atmosphere by a line which bounds a black area on the diagram. The scale of daily supply of energy in kilowatt-hours per square dekametre is marked at the side. The area between the black and the base line represents the total energy available in a year and is given in figures near the right hand of the base-line. Below the black area is another line which marks the total amount of energy received from sun and sky as recorded on a pyranometer or Callendar recorder. This is the item which is noted as B in the balance-sheet. The daily amounts are shown as averages for months. The white area between the shading and the base-line represents the total amount of energy received at the station in the course of a year. The figure is given at the left-hand end of the base-line.

The area between the receipt-line and the black, which is shaded grey, represents the part of the original energy which is lost by reflexion into space as part of the earth's albedo, the item D of the balance-sheet, together with item C the part which is absorbed by the atmospheric constituents, carbon dioxide, water-vapour and dust.

The information derived from the pyrheliometer is set out by lines within the area of total receipt. They separate the direct solar radiation from the total and so isolate, as the upper part of the white area, the amount received from the sky. It will be seen that in the winter months, when the sun's altitude

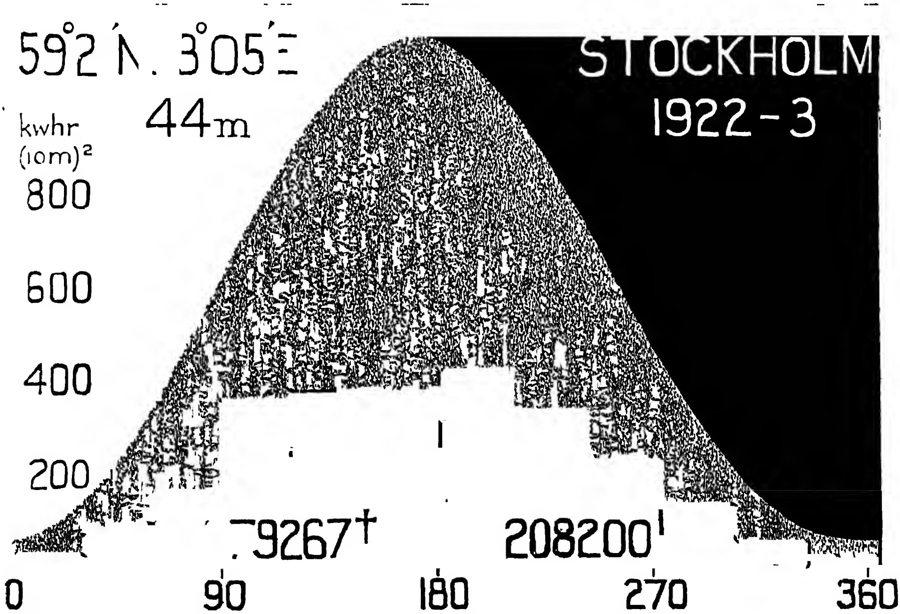


Fig. 61. Curve of daily totals of solar radiation on a horizontal surface outside the atmosphere in the latitude of Stockholm, with monthly totals of sun and sky-radiation and of sun-radiation alone, as measured on Ångström's pyranometer. The portion due to sky-radiation alone is represented by the intermediate part of the diagram.

at noon is very small, the greater part of the radiation which is received upon a horizontal surface comes from the sky. The figures[1] upon which the column-graph is based are as follows:

Ratio of sky-radiation to the total (sun and sky) radiation expressed as percentage

Stockholm, July 1922 to June 1923

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
79	74	42	35	33	35	26	36	45	54	77	97

In a more recent publication[2] Ångström gives the following figures as representing the average monthly values for the period 1905 to 1926:

63	56	43	27	23	24	19	26	34	52	77	87

[1] *Q.J. Roy. Meteor. Soc.* vol. L, 1924, p. 123.
[2] *Meddel. från Stat. Meteor.-Hydrog. Anst.* Band 4, No. 3, Stockholm, 1928, p. 21.

There is a large amount of information of one sort or another about sky-radiation, but it is not generally presented in a form in which it can be directly compared with the total radiation from sun and sky. We shall endeavour to give some account of it in order that the reader may not find himself hampered for lack of it in the further prosecution of the study of radiation as an agent in developing the sequence of weather; but let us first dispose of the information about the total radiation upon a horizontal surface.

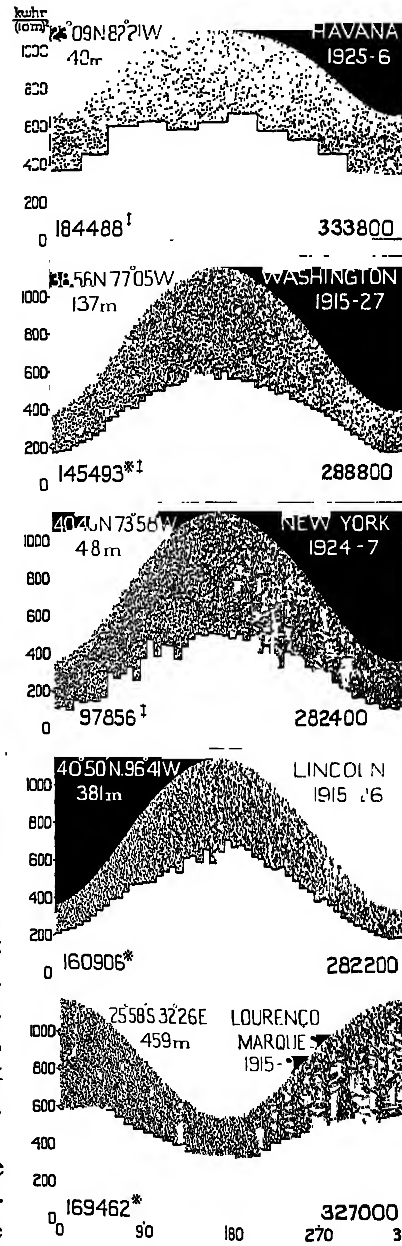
Following the same general scheme as that adopted in fig. 61 for Stockholm we give diagrams for 10 stations in various parts of the world, figs. 62 and 63, which show by the black curtain the amount of solar radiation outside the atmosphere, and by the unshaded portion the amount received on a horizontal surface; the shaded portion between the two represents the part which is lost by reflexion or absorption in the atmosphere, the items C and D.

The geographical coordinates of the station, its name and the duration of the observations represented are marked on the black curtain.

It will be seen that the available fraction A of the total possible energy is different for the different stations. The order in percentage is: Lincoln 57, Havana 55, Johannesburg 54, Lourenço Marques 52, Washington 50, Toronto 39, New York and Rothamsted 35, South Kensington 33 and Chicago 32.

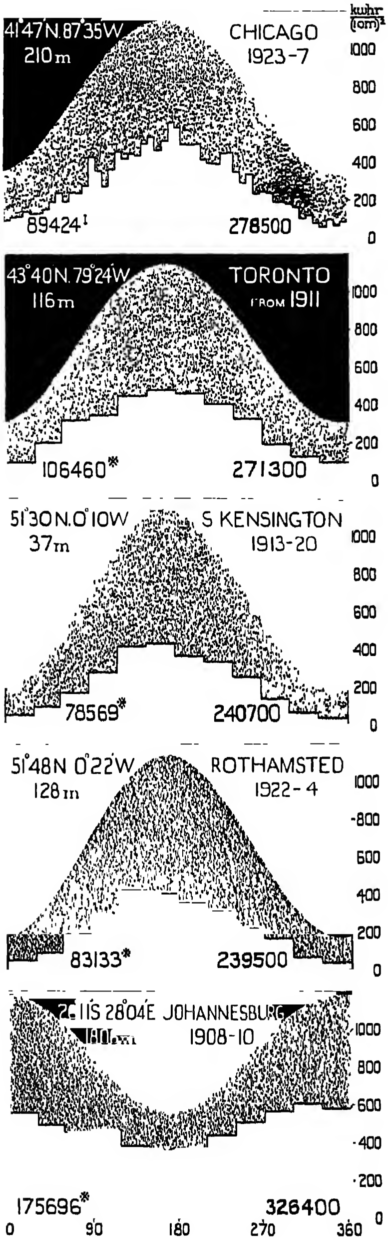
The information is derived from the report of the meeting at Madrid, supplemented by figures given by Kimball in the *Monthly Weather Review* for April 1927. It should be remarked that the totals which we obtain from the diagrams do not in all cases agree with totals quoted

FIG. 62. TOTAL RADIATION OF SUN AND SKY



The symbol attached to the figure for the total energy received indicates the instrument which has been in use, * means "Callendar," ‡ "Weather Bureau thermoelectric."

FIG. 63. TOTAL RADIATION OF SUN AND SKY



The annual aggregate in kw-hr/(10 m)² of radiation received is given on the left-hand side, and of that incident upon the atmosphere on the right.

by Kimball. In forming the percentages we have adopted Kimball's figures as being probably nearer the original values. The cases where the difference is appreciable are Chicago which we should reckon as 37, and New York 40.

INDIRECT SHORT WAVES

We have pointed out that a pyranometer records the gross total of the item B for any station. It includes the direct radiation which was represented for a number of stations in the tables of pp. 122–4 and the sky-radiation. Both these contributions to the energy of the atmospheric circulation are absorbed or at least absorbable by the ground. Whether in the future, when we come to consider the effect of radiation upon weather, it will be necessary to treat them as distinct or as combined we are at present unable to say. But little attention has been paid to that aspect of the general question of radiation while a great deal has been paid to the variation in the amount of radiation recorded, and results have been obtained which in themselves are at least interesting.

With regard to this kind of information H. H. Kimball remarks:

> A comparison of the two curves for Sloutzk [viz. that for direct solar radiation and that for direct solar and diffuse sky-radiation] and also that for Arosa with the curve for near-by Davos, indicates the very considerable part of the total solar thermal energy that is received diffusely from the sky, amounting in many months to 50 per cent. On the other hand, the curve for Lindenberg shows nearly as much energy received from the sun on a surface normal to its rays as the total energy received on a horizontal surface from the sun and sky at Davos, which is at a lower latitude and higher altitude.
>
> (*Monthly Weather Review*, vol. LV, Washington, 1927, p. 156.)

It would appear therefore that so far as a horizontal surface is concerned the supplementary radiation from the sky in the course of a day is more than counterbalanced by the loss through the cosine-effect of the solar altitude as compared with radiation at normal incidence.

The reduction of the influence of direct solar radiation upon a horizontal surface in northern latitudes may be illustrated by the results which are given for Sloutzk (Pavlovsk). They are summarised in fig. 64 which shows the line of sunrise and sunset as the boundary of the black, a curve for the hours of intensity 20 kw/(10 m)^2 in different months, another curve of hours of solar altitude 30° and within both curves the maximum values of solar radiation about midday in the summer months. The smallness of the amounts is due partly to the obliquity of the surface in relation to the sun's rays at 60° and partly to local atmospheric absorption.

SLOUTZK, 60° N, 30° E, 16 m, 1913–19

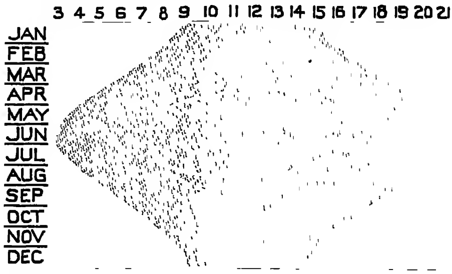


Fig. 64. Diurnal and seasonal variation of the solar radiation received on a horizontal surface in kilowatt-hours per square dekametre. Crova-Savinoff instrument.

The broken line shows when the sun's altitude is 30°.

Some of the highest hourly values are shown by figures.

Cloudless days

Figures for the sky-radiation from a cloudless sky as a percentage of the total radiation received on a horizontal surface are given in the following table[1]:

(Sky-radiation as percentage of total radiation)

Solar altitude / Station (height)	82·5°	65°	60°	41·7°	35°	30°	23·5°	19·3°	16·4°	14·3°	12·6°	11·3°	5°
Washington (137 m)													
Winter	—	—	—	12	—	16	20	23	25	29	32	37	—
Spring	—	10	—	13	—	17	20	24	28	32	35	40	—
Summer	—	19	—	21	—	24	27	31	34	37	38	40	—
Year	—	16	—	17	—	20	23	25	29	33	36	38	—
Lincoln (381 m)	—	15	—	16	—	19	21	24	27	30	33	36	—
Madison (308 m)	—	16	—	16	—	24	25	26	34	36	—	—	—
Hump Mt (1500 m)	—	—	—	8	—	10	12	13	15	16	18	19	—
Mt Wilson (1737 m) 1913	14	14	14	16	20	20	24	27	—	32	—	38	55

A comparison of the observations at 4420 m (Mt Whitney) and at sea-level (Flint Is.) for the same solar altitude, 65°, gives sky-radiation as 8 per cent. of the total at the high-level and 19 per cent. at sea-level.

In order to keep in touch with actual magnitudes we give the figures for Mount Wilson in g cal/(1 cm^2 min) and kw/(10 m)^2. The observations were

[1] H. H. Kimball, *Monthly Weather Review*, vol. LV, Washington, 1927, p. 156.

made in 1913 when the atmosphere may still have been hazy after the eruption of Katmai:

	Solar altitude						
	82·5°	65°	47·5°	35°	25°	15°	5°
	g cal/(cm² min)						
Sun	1·507	1·355	1·041	·780	·524	·233	·046
Sky	·240	·226	·205	·189	·162	·110	·056
Total	1·747	1·581	1·246	·969	·686	·343	·102
	kw/(10 m)²						
Sun	105·0	94·4	72·6	54·4	36·5	16·2	3·2
Sky	16·7	15·8	14·3	13·2	11·3	7·7	3·9
Total	121·7	110·2	86·9	67·6	47·8	23·9	7·1

Even with a cloudless sky the ratio of sky-radiation to total radiation varies considerably on different occasions. At Hump Mountain (lat. 36° 8′ N, long. 82° 0′ W, 1500 m) the range of values of the intensity of sky light on a horizontal surface at a time when the sun's altitude was 30° ran from ·060 to ·130 g cal/(cm² min), with a mean of 0·0796.

For Calama (800 metres higher but in a rainless district) the intensity on cloudless days is nearly the same as at Hump Mountain, from ·060 to ·113 g cal/(cm² min), with a mean of ·0757.

For solar altitude 19° (air-mass 3) the values of total sky brightness on a horizontal surface at Calama vary from ·053 to ·108 g cal/(cm² min), with a mean of ·0642. (Summarised from *Annals of the Astrophysical Observatory*, vol. IV, 1922, pp. 261–2 and p. 274.)

The variations in the sky-radiation for different solar altitudes, expressed as a percentage of the total radiation, on smoky days and on an unusually clear day are given by Kimball[1] in the following figures for Mount Weather arranged according to air-mass from the minimum to 1·5 and by steps of ·5 to 4·5:

1914	Solar altitude 70·8°	41·7°	30·0°	23·5°	19·3°	16·4°	14·3°	12·6°	Sunrise to noon
Smoky days			Sky-radiation as percentage of total						
May 20	23	31	38	48	51	53	64	67	32
May 26	26	35	42	48	55	63	70	71	36
Unusually clear									
June 30	9·8	10	13	16	20	23	27	29	12

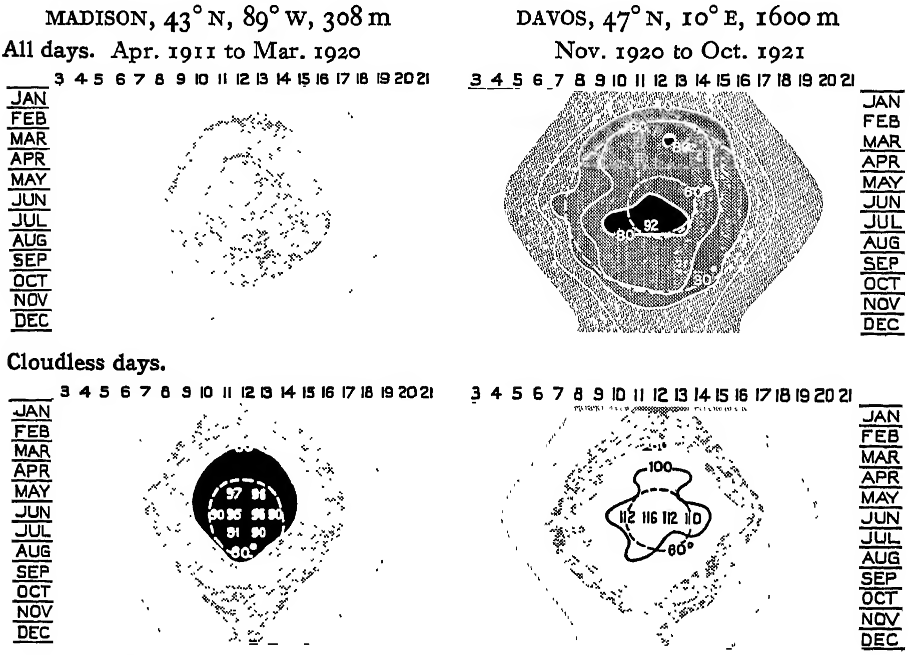
The effect of cloudiness on total radiation

Whatever the ultimate requirements may be it is only natural for those who are busy with solar radiation to draw a distinction between clear days and cloudy days, and to consider that meteorologically the effect of the two must be different. In the results obtained from the records of Callendar instruments or other pyranometers and represented in figs. 61–63 all days have been included. The irregularity of the sequence of daily values on the Callendar record as shown in fig. 60 suggests doubt as to the propriety of covering such differences by a single mean value, and to some extent that point has been met for certain stations at which the values for cloudless days have been

[1] *Monthly Weather Review*, vol. XLII, Washington, 1914, p. 310.

obtained separately for comparison with the values for all days. We have figures of this type for Davos on the eastern side of the Atlantic and for Madison in Wisconsin on the western side (figs. 65–68).

It will be seen that the cloudless days provide much greater radiation and much more "gradient of radiation" in the part of the diagram which represents the higher solar altitudes. The difference is less marked in the early morning or late afternoon. This leads us to recall Kimball's remark that a cloudless day does not provide an effective classification. A gradual change in the amount of



Figs. 65–68. Diurnal and seasonal variation of the total radiation received on a horizontal surface in kilowatt-hours per square dekametre. The upper pair of diagrams represent observations on all days, the lower pair those on cloudless days only. The broken lines show the points when the sun's altitude is 30° and when 60°. Some of the highest mean hourly values are shown by figures.

radiation received on a horizontal surface as the solar altitude changes even on a cloudless day is not of course a matter for surprise: on account of the increased "air-mass" the same thing happens also as we have already seen when the direct insolation alone is measured. We must be prepared to acknowledge that owing to the nebulosity of dust and other material particles there are great differences in the transparency of the atmosphere between different days which are classed as cloudless on account of the absence of "cloud."

In England certainly, and possibly elsewhere, the cloudless yet nebulous condition of the atmosphere due to dust or some nucleation that could not be called cloud is well known; in days gone by it was sometimes called a "blight."

It might quite well be regarded as the precursor of a thunderstorm; we are not aware of any study of the relation of summer thunderstorms to the previous state of the atmosphere in respect of radiation, though the information cannot be far to seek.

Kimball has treated the effect of cloudiness upon the measures of total radiation in a general manner by confronting the records of the Callendar instrument at Washington with the recorded observations of cloud-amount and of the percentage of possible sunshine.

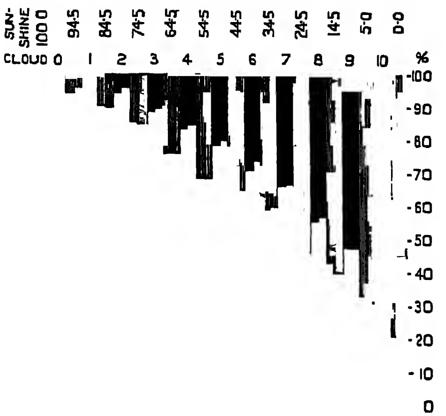


Fig. 69. The diagram shows the total radiation at Washington as a percentage of that which would be received if the sky were clear. The black columns indicate the amount of radiation blocked out by corresponding cloud-amounts 1, 2, 3,... 10, and the columns with vertical hatching the radiation blocked out on the days with percentage of possible sunshine 94·5, 84·5,...14·5, 5·0 and 0·0. In both cases the left-hand column refers to the winter months, October to March, and the right-hand to the summer months, April to September.

From the Callendar records the average daily amount of radiation for each decade (except that for June and December the averages are for the entire month) has been determined for days on which the cloudiness was recorded as 0, 1, 2, 3, etc., to 10 respectively, and also on days for which the percentage of possible sunshine as recorded by the Marvin sunshine recorder was 100, 99 to 90, 89 to 80, etc. to 9 to 1 and 0, respectively. From these decade and monthly averages the seasonal and annual averages which are expressed as a "percentage of clear-sky radiation" have been derived. The seasonal differences are not important....From this latter [the annual averages] it is seen that with the daily cloudiness recorded 10 the radiation averages about 29 per cent. of clear-sky radiation. This is a little greater than for zero percentage of possible sunshine, namely, 22 per cent., for the reason that the sun may sometimes shine with the sky more than 95 per cent. covered with clouds. Also, 50 per cent. clear-sky radiation intensity corresponds to an average cloudiness of 9, and to a percentage of possible sunshine of 20; and 50 per cent. possible sunshine corresponds to 71 per cent., and 5 cloudiness to 82 per cent. of clear-sky radiation intensity. In general, the percentage of possible hours of sunshine is greater than the cloudiness would lead us to expect, the maximum difference occurring when the sky is rather more than half covered with clouds.

(H. H. Kimball, *Monthly Weath. Rev.* vol. XLVII, Washington, 1919, p. 780.) The quotation refers to mean values for three stations.

From Kimball's figures we have made a diagram which is reproduced in fig. 69 and shows the observed radiation on the selected days as percentage of clear-sky radiation. Kimball divides his observations into summer months, April to September, and winter months, October to March, and the distinction has been preserved in the diagram.

Clouds as sky-radiators

If we pass from these general results to a more detailed consideration of the effect of clouds on the radiation received at the ground we may note first that Abbot[1] has placed on record the figures which correspond with the natural impression that even in clear sky the radiation from the immediate neighbourhood of the sun is greater than that from the more distant zones.

In a paper in the *Monthly Weather Review* for August 1914 Kimball gives an interesting diagram which shows the effect of interposing a screen between the receiver and the sun, with a curve which illustrates what we may call the "sunshade effect" at Mount Weather. An illustration (fig. 70) of a Callendar record on a day of passing clouds shows how sensitive the recorder is to the effect of cloud, which is not very different from sunshading.

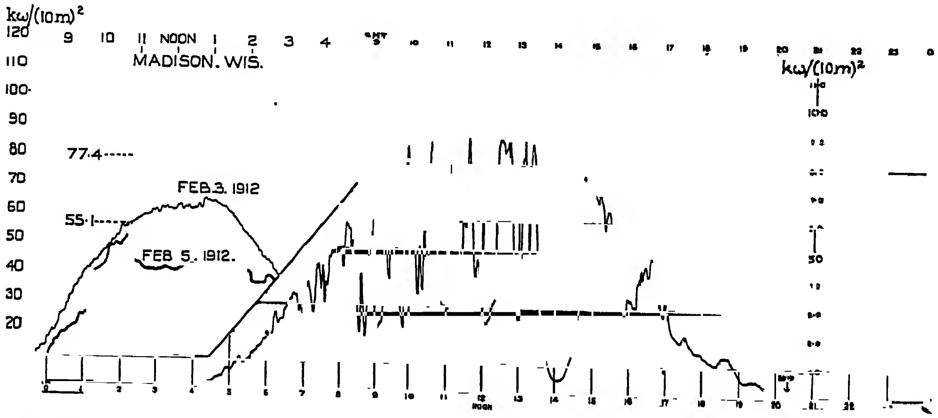


Fig. 70. Record of the total radiation received on the horizontal surface of a Callendar pyranometer at South Kensington on a day with passing clouds, 23 June 1926. Inset: Records from a similar instrument at Madison, Wis., on a perfectly clear day, 3 February 1912, and on a day with a bright sheet of alto-stratus, 5 February 1912.

Kimball has a note to the effect that the total radiation as recorded upon a pyranometer (or Callendar recorder) may actually be increased temporarily by the presence of clouds near the sun, a fact which is illustrated (inset to fig. 70) by the remarkable peak at 10 h 40 in the record for Madison of 5 February 1912. It was attributed to the approach of a glaringly bright sheet of alto-stratus cloud.

To summarise, the records from Callendar recording pyrheliometers show that with favourable conditions of sun and clouds the radiation...received from the sun and sky may be at least 40 per cent. in excess of what would have been recorded had the sky been free from clouds, and that an excess of 10 per cent. is quite common. In consequence, on partly cloudy days, such as June 17, 1912, at Madison, Wis., the cloudiness may diminish but slightly the total amount of radiation received at the surface of the earth. The concentration of the solar rays by clouds is principally due to reflection from their surfaces.

[1] *Annals of the Astrophysical Observatory*, vol. IV, 1922, p. 263.

While some heat rays always penetrate through clouds, in the case of dense thunderstorm-clouds the amount may be less than 1 per cent. of the radiation-intensity at noon when the sky is clear.

(H. H. Kimball and E. R. Miller, 'The influence of clouds on the distribution of solar radiation,' *Bulletin of the Mount Weather Observatory*, vol. v, 1912, p. 168.)

Pursuing the subject to the radiation diffused from the clouds, the records upon which figs. 61–63 are based include occasions on which the sun was screened by cloud as well as those when the cloud covering a fraction of the sky left the sun unscreened. In the case of the radiation from the sky alone we have the following information as to the effect of clouds from A. Ångström's[1] observations in summer at Upsala in 1918 and Washington in 1919. The observations were made when the sun's altitude was between 60° and 80°.

	g cal / cm² min	kw/(10 m)²
Radiation from clear sky:		
with direct solar radn. about 0·75 g cal/cm² min, 52 kw/(10 m)²	0·10	7
" " " 0·50 g cal/cm² min, 35 kw/(10 m)²	0·30	21
Sky covered by Ci-st	0·15 to 0·30	10 to 21
Sky covered by A-st	0·20 to 0·40	14 to 28
Sky covered by St-cu (not very dense)	about 0·50	35
Sky covered by Ni (not very dense)	about 0·35	24
Sky covered by Ni (very dense)	0·10	7

The observations are interpreted as showing that, apart from direct sun-radiation, cloudiness increases the amount of heat received upon a black body at the surface so long as the cloudiness is due to comparatively light clouds such as Ci or A-st, whereas the heavy clouds Ni and Cu-ni cause a decrease of radiation, the effect of the intermediate cloud-layer St-cu may be either way. As we have already noted, the effect of a thunder-cloud upon the Callendar record is practically to stop radiation altogether.

Ångström concludes:

For the cloudiness corresponding to the maximum of sky-radiation, the sun radiation is practically nil. The radiation income corresponding to the cloudiness 10 is consequently under these conditions not equal to 0, as is often assumed, but about 50 per cent. of the sun radiation when the sky is clear. On the average the cloudiness 10 causes a decrease in the total heat income down to about 30 per cent.

A. F. Moore and L. H. Abbot reached similar conclusions from observations at Hump Mountain:

Taking ·0700 calories [4·9 kw/(10 m)²] as a fair average of the intensity of the radiation on a horizontal surface from a cloudless sky at hour angles of the sun of 3 to 4, it will be seen that for cloudy skies the values are from four to nine fold for average clouds, and from one to four fold for very heavy clouds. Very often, just preceding the precipitation of rain, the radiation drops very considerably and very rapidly.

With low fog the observations unfortunately are few, but the indications are that the radiation is ten-fold or more that of clear skies.

An average cloudy sky, if the clouds are not too thick, lets through about as much radiation (measured on a horizontal surface) as do the sun and a clear sky combined with the sun at an altitude of about 15°. The radiation from a low fog is about the same as from the sun and a clear sky at 30° sun.

(*Smithsonian Miscellaneous Collections*, vol. LXXI, No. 4, Washington, 1920.)

[1] 'Some problems relating to the scattered radiation from the sky,' *Monthly Weath. Rev. Washington*, vol. XLVII, 1919, p. 797.

As a further illustration of the variation in the amount of radiation received by diffuse sky-radiation when no sunshine is registered, we have the following note on observations at South Kensington. On the days when no sunshine was recorded in the year 1920 the indications of the Callendar radiation recorder ranged from 2 kw-hr/(10 m)^2 for a day in November to 236 kw-hr/(10 m)^2 for a day in July.

Sky-searching for short waves: Mr Dines's observations

The short-wave radiation from different parts of the sky is included in the investigation by W. H. Dines in the application of the instrument which we venture to call a sky-searcher. It gives the equivalent black-body temperatures of different parts of the sky and thence by calculation the total amount of radiation which is received upon a horizontal surface. For the investigation of short waves the long-wave radiation is cut out of the measurement by interposing a glass plate in the path of the search-beam.

Dines's[1] instrument deals with heat-radiation somewhat in the same way as a searchlight deals with light. It was based originally upon a design of L. F. Richardson. Near the closed inner end of a horizontal metal cylinder of which the outer end projects horizontally from the side of a large tank of water, is placed a pile of thermo-junctions of copper-eureka, arranged as thin disks with thin connecting strips; alternate disks are edgewise on, with the intervening ones broadside on. Opposite to the open end of the immersed cylinder is a spherical mirror of silver or nickel with its axis at 45° to the axis of the cylinder. If the thermopile had emitted a luminous beam like a searchlight the rotation of the mirror about the axis of the cylinder would have caused the beam to sweep the sky in a vertical plane. Underneath the mounting of the mirror, sunk in the ground, was a vertical pit formed by a drain-pipe, the upper end open; the lower part contained water, a nearly perfect radiator in the vertical direction, at nearly invariable temperature.

A searchlight thus handled would show a bright patch upon any surface *less bright than its own beam*; the sky-searcher, in like manner, shows a loss of its energy, by the movement of a galvanometer, if the portion of the sky or the water in the ground, when covered by its beam, is at a lower temperature than the pile. Conversely if the water in the pit, or the cone of atmosphere covered by the beam, is effectively "warmer" than the pile the opposite deflexion of the galvanometer (properly adjusted) makes the difference apparent and registers the amount. The effective warmth of the atmosphere depends upon its radiative capacity as well as its temperature.

In this way the "radiation-temperature" of any object covered by the beam, of the water in the ground, of the surface of the meadow in which the instrument is placed or of the heterogeneous radiating material of the atmosphere included in a beam directed to the sky, can be measured; the amount which is being radiated from a square centimetre of the water in the pit can be computed by Stefan's law from the known temperature of the water, since the

[1] *Geophysical Memoirs*, No. 18, M.O. publication 220 h, London, 1921.

water can be regarded as a perfect radiator. Regarding the radiation from the field or from the sky, whether cloudy or not, the instrument gives by simple calibration, in like manner, the temperature of a black body which would give the same amount of radiation as that which is coming from the field or the sky.

The other bodies, namely the thermopile and the water-pit, being at constant temperature, the invisible beam of the instrument registers *the black-body temperature equivalent to the radiation of the part of the ground or sky covered by the beam* whether it is grass or cloud or clear air. The amount of radiation which the grass or cloud or clear air is emitting may be expressed as that which would be emitted according to Stefan's law by one square centimetre of black body at the specified temperature. But in fact we need hardly calculate the amount of radiation involved, the equivalent temperature of the radiator is enough.

If the equivalent temperature of a portion of the sky is read as $n$tt, the amount of radiation received is expressed as σn^4 and that is also the amount which a square centimetre would distribute over a hemisphere of which it was the centre. That would in fact be balanced by incoming radiation from a hemispherical enclosure of the same temperature. The inflowing radiation as determined by Dines is that which would come to a horizontal square centimetre from an enclosure of the same temperature as the equivalent temperature of radiation.

Dines's conclusions about diffuse solar (short-wave) radiation are as follows:

1. The amount coming from the neighbourhood of the zenith on a clear day in gramme calories per square centimetre per day is approximately equal to the number expressing the altitude of the sun in degrees.

2. The amount increases to a maximum as the zenith angle increases to a value of about 60°, at which angle the maximum occurs.

3. A grass field reflects about one-third of the diffuse solar radiation from the sky that falls upon it.

4. Broken clouds, showing much white, reflect the most radiation; in the midday hours in September the amount may reach 300 g cal [14·5 kw/(10 m)^2]. It does not seem to matter if the clouds are high or low; fog, with the sun just breaking through, will show a large value. As with clear skies, the amount increases with the zenith distance, but the values from any definite direction are subject to rapid changes.

5. Especially dense and heavy cloud sheets supply about the same diffuse solar radiation as a clear sky does. A dense fog supplies about as much as a sheet of cirro-cumulus.

6. On cloudless days low haze adds to the diffuse solar radiation.

7. The direction of the sun has very considerable effect on clear days. The radiation from parts of the sky near the sun is the greater, but the observations do not suffice to lay down any fixed rule.

The following figures give the means for all the observations in October that were taken within four hours of midday, in November within three hours, and in December within two and a half hours.

Measures in g cal/cm^2 day

Alt. of zone	82° 30′	67° 30′	52° 30′	37° 30′	22° 30′	7° 30′	Grass field
Oct.	57	60	70	75	78	66	35
Nov.	51	50	55	56	57	40	22
Dec.	41	41	46	47	52	40	17

At and after sunset short-wave radiation from the sky is inappreciable.

An analysis of the results of observations of the same kind extended over the five years 1922 to 1926 is given in the following tables[1].

Short-wave radiation from cloudless and overcast skies

Measured between 10 h and 14 h in winter and between 9 h and 15 h in summer in gramme calories per square centimetre per day convertible to kilowatts/(10 m)^2 by the factor 0·0484

Cloudless skies

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Radiation from celestial hemisphere	26	30	37	51	71	56	46	39	45	41	26	22	41
From grass-field	22	19	50	82	94	69	86	38	71	53	28	18	52
Number of observations	17	14	9	16	13	16	7	3	14	21	19	19	168

Overcast skies

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
From hemisphere	34	44	72	72	109	120	128	120	106	101	43	24	81
From grass-field	9	10	17	18	30	30	28	26	20	19	10	5	19
Number of observations	26	17	19	17	19	29	14	20	19	23	20	29	252

The average zonal distribution for the whole year is as follows:

Altitude of zone	82½°	67½°	52½°	37½°	22½°	7½°	Grass-field
Cloudless skies	34	35	37	40	48	58	52
Overcast skies	93	95	90	81	69	39	19

Analysis of the effect of the atmosphere on the radiation from the sun

Item D of the general balance-sheet. The earth's albedo

The amount of radiation which is received from sun and sky at any station in the course of the year is by no means the whole of that which reaches the confines of the atmosphere, and which would enrich the surface if the air were always perfectly transparent. That is far from being the case. Wherever there are clouds a large fraction of the radiation, probably about three-quarters of the whole, is diffused by reflexion or scattering, and passes out into space. The same is true for the areas which are covered with snow or ice. These different forms of water account for the chief part of what is known as the earth's albedo which corresponds with moonlight for the moon, or the diffused light of the sun by which the planets are visible.

Those parts of the earth's surface which are not screened by clouds or covered by snow do not absorb the whole of the radiation which reaches them, part is reflected, diffusely or otherwise, by the surfaces of water, earth or vegetation, and makes its contribution to the radiation which is retransmitted to space as short waves. These contributions from clouds and from snow and from other parts of the surface, combine to form the item which, for the whole earth, is estimated as item D of the balance-sheet.

[1] 'Monthly mean values of radiation from various parts of the sky at Benson, Oxfordshire,' by W. H. Dines and L. H. G. Dines. *Memoirs of the Royal Meteorological Society*, vol. II, No. 11, London, 1927.

Some of the radiation which traverses the atmosphere is absorbed by the dust, the water-vapour or the carbon dioxide of the air and raises its temperature and consequently increases the long-wave radiation from the body of the atmosphere itself. It is brought to account in the balance-sheet with items E and F.

W. H. Dines estimated the general albedo of the earth, that is the return to space of short-wave radiation, from the sea, the clouds, the air and the varieties of surface of the earth, at 50 per cent. of the incident radiation. The figure for the ratio of energy reflected from different parts to the energy of radiation incident upon them may range from 78 per cent. for clouds or snow to 10 per cent. or less for the surface of water. What the actual value in the case of any particular material may be is a matter to be decided by observation. Nevertheless in this as in other branches of meteorology it is helpful to have a provisional estimate for reference while the final standard is being gradually evolved.

After elaborate examination of the available data C. G. Abbot[1] arrived at the value of 37 per cent. as the equivalent albedo of the earth as a whole, of which reflexion from clouds accounts for 29 per cent., reflexion from the earth 2, and reflexion from the air 6. A more recent computation by Aldrich based on the value 78 per cent. as the reflecting power of clouds gives the value 43 per cent.

A pyranometer suspended below the basket of an Army observation balloon was used to measure the reflecting power of a level cloud-surface practically filling a hemisphere of solid angle. Over 100 determinations were made. The solar air-masses ranged from 2·8 to 1·2 and the sky above was cloudless and very clear. A mean value of 78 per cent. is obtained. No change of total reflection depending on solar zenith distance is apparent within a range of zenith distance from 33° to 69°.

(*Ann. Astrophys. Obs.* vol. IV, 1922, p. 381.)

The details of the computation of the albedo have been reconsidered by G. C. Simpson in *Memoir* No. 23 of the Royal Meteorological Society.

In the balance-sheet we have taken no account of energy received from other external bodies than the sun. Energy is in fact received from the moon, planets and stars, otherwise we should not see them, but the amount received is too small to affect the measurements which express the condition of the atmosphere.

For the purpose of comparison we give the figures estimated by astronomers for the light reflected from the moon and planets.

The visual albedo of the moon and planets expressed as percentage of the sunlight incident upon them

(From *Smithsonian Physical Tables*, 7th edition, 1920, p. 417)

Moon	7·3	Earth	43	Saturn	63
Mercury	6·9, 5·5	Mars	15·4	Uranus	63
Venus	59	Jupiter	56	Neptune	73

Clouds are visible on Jupiter as the photographs on pp. 169–71 of vol. II show. The brilliancy of Venus is regarded as due to the planet being surrounded by an envelope of cloud.

[1] *Ann. Astrophys. Obs.* vol. II, 1908, p. 163.

The reflexion of solar energy by different surfaces

So far as possible the measures of the intensity of the incident radiation are derived from instruments which make use of a perfectly black receiving surface; few material surfaces approach perfection in that respect; all reflect, whether as regular or diffuse reflexion, some fraction of the incident energy.

Expressed as percentage of the incident radiation the figures for reflexion from various natural surfaces are as follows[1]:

	Per cent.		Per cent.			Per cent.
Rock	12 to 15	Wet sand	9	Snow		70 to 80
Dry mould	14	Grass	10 to 33	White sandstone	At 20°	24
Wet mould	8 to 9	Water, sun 47°	2	Clay marl	inci-	16
Grey sand	18	Water, sun 5½°	71	Moist earth	dence	8

It is to be remembered that the energy which is incident must be accounted for either as reflected or absorbed or transmitted. Disregarding what in certain circumstances may be transmitted the differences between 100 and the percentages given above represent the portion of the incident energy which is absorbed. The general result is thus summed up:

> When the ground-surface is not snow-covered reflexion is insignificant. Black soil, areas covered by pine and spruce forest, or hardwoods not in leaf reflect but a small per cent. of the radiation incident on them; grassland, hardwoods in leaf and growing crops can reflect 15 per cent., while dry sand and light coloured rocks can send back 30 per cent. of the insolation which they receive. The reflexion from a snow cover however is over 70 per cent.
>
> (H. I. Baldwin, *Bulletin of the American Meteorological Society*, 1925, p. 123.)

The energy which returns to space from the surface in this way is, properly speaking, included in the 50 per cent. of the incident radiation that is allowed in the balance-sheet as albedo; but the contribution is not of great importance except in the case of a surface of snow. The reflexion from the water-surface of the earth has been estimated[2] at 6 per cent., that of the remainder, including the snow and ice of the Arctic and Antarctic regions, at 15 per cent., and that of the whole earth's surface at 8 per cent.

These figures are understood to refer to the short-wave radiation which is received from the sun.

The energy which is absorbed at the earth's surface raises the temperature of the absorbing material and is returned to the atmosphere either by conduction and convexion, item L, or as long-wave radiation, item G.

Of the substances enumerated, water and green leaves have the power of transmitting as well as reflecting and absorbing. For the green leaves of trees Ångström estimates that in early summer when leaves contain much water reflexion accounts for 19 per cent., absorption 55·5 and transmission 25·5, but in late summer when leaves are drier the figures become 29, 38, 33.

[1] See Ångström, 'The albedo of various surfaces of ground,' *Geografiska Annaler*, 1925, p. 323; F. W. P. Götz, *Das Strahlungsklima von Arosa*, Berlin, 1926; Zöllner, quoted in *Annals of the Astrophysical Observatory*, vol. II, p. 161, also L. F. Richardson, *Proc. Roy. Soc.* A, vol. XCVI, 1919, p. 25 and Report on photometers, *Met. Sec. U.G.G.I.* Mem. 2.

[2] *Annals of the Astrophysical Observatory*, vol. II, 1908, pp. 161–2.

THE ANALYSIS OF RADIANT ENERGY ACCORDING TO WAVE-LENGTH

We have so far regarded radiation as divisible into two different kinds, namely, short-wave radiation from the sun and long-wave radiation from the earth as represented in fig. 49. In accordance with Wien's law we may attribute this mode of differentiation to the fact that the temperature of the sun's surface, estimated at about 6000tt, far exceeds the temperature of the air or of any part of the natural surface of the earth, something between 320tt and 200tt. The wave-length of maximum radiation in a sunbeam is about $\cdot 5\mu$. It is so far removed from the maximum of radiation for terrestrial objects (between 8μ and 12μ) that on a gradually increasing scale of wave-lengths, or gradually diminishing frequency, the intensity of solar radiation has died out before the effective wave-lengths of the terrestrial radiation have been reached.

The range of radiation at experimental temperatures can be illustrated by the well-known curves of fig. 71 which are based on measurements of Lummer and Pringsheim[1]. The energy of the radiation of a black body at 287tt only begins at 4μ, reaches its maximum about 10μ and can be regarded as extending beyond 60μ, but with very little intensity beyond 30μ. The limits of visibility of the solar spectrum are from $\cdot 4\mu$ to $\cdot 8\mu$. Thermal energy can be traced from $\cdot 2\mu$ to beyond 12μ but it is not really appreciable beyond 3μ. For the solar spectrum outside the atmosphere Abbot[2] gives the following distribution for different parts of the spectrum in decimal fractions of the whole energy:

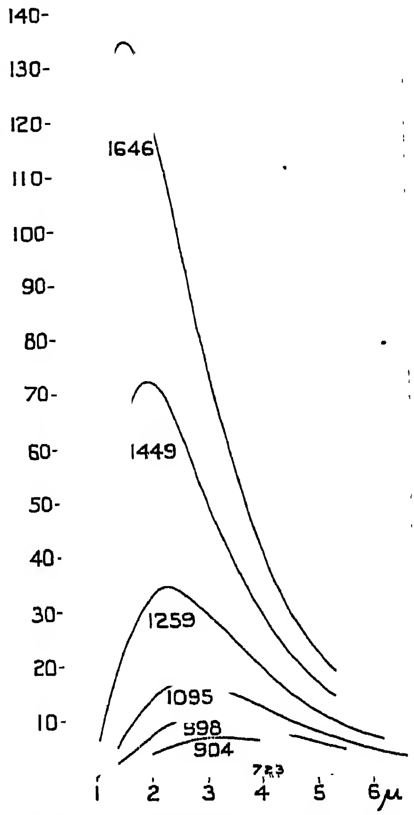


Fig. 71. Distribution of intensity of radiation among the wave-lengths, of the radiation of a black body at different temperatures (Lummer and Pringsheim).

The ordinates are intensities or emissive powers (which are proportional) and the abscissae are wave-lengths. The total energy of radiation for a given temperature is represented by the area between the curve and the horizontal axis. This area increases according to the fourth power of the absolute temperature according to Stefan's law. The scale of ordinates represents millions of c, g, s units per micron.

Range of wave-length μ	0–0·45	·45–·55	·55–·67	·67–·90	·90–1·10	1·1–1·4	>1·4
Energy fraction of total	·12	·20	·17	·20	·11	·08	·12

[1] 'Kritisches zur schwarzen Strahlung,' *Ann. der Physik*, Vierte Folge, Bd. VI, Leipzig, 1901, p. 200.

[2] *Annals of the Astrophysical Observatory*, vol. II, 1908, p. 128.

The laws of absorption and of scattering

We have next to recognise that in the transmission of radiation, whether long wave or short wave, between the sun and the earth, the atmosphere intervenes as an imperfectly transparent medium. In illustration of the effect of the atmosphere upon the short-wave radiation we may refer to fig. 72, which reproduces the normal curve of relation of radiation to wave-length obtained by S. P. Langley[1] with the bolometer. The notable departures from the smooth run of the curve are due to the absorption of the atmosphere. They amount altogether to about 10 per cent. of the incident radiation.

The imperfection of its transparency arises from two causes, first the absorptive effect of certain constituents of the atmosphere, viz. water-vapour,

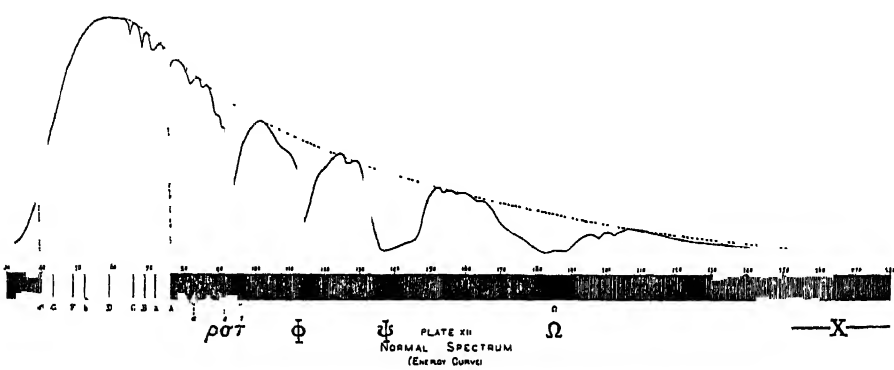


Fig. 72. Normal solar spectrum, bolometric curve of energy referred to a scale of wave-lengths in terms of millionths of a centimetre, showing the absorption of the atmosphere (S. P. Langley).

carbon dioxide and ozone, and secondly the disturbing effect, upon transmission, of the molecules of the air itself, or of the liquid particles (with or without solid nuclei) which assemble themselves together in clouds, and the fine solid particles of salt, sand, grit or soot, which are carried for vast distances from volcanoes, forest-fires, seashores, deserts or chimneys.

These are always present in the atmosphere to a greater or less degree and produce such visible effects as the brilliance of an illuminated cloud, the darkness of a sandstorm, the yellow or crimson colours of dawn and sunset, and the red colour of the sun in city fog.

In the solar radiation as registered by the bolometer, besides the loss due to the scattering of light by the molecules of air there is notable absorption in the region of ultra-violet below $\cdot 4\mu$ which is attributed to ozone. There are also absorption bands which are indicated by ρ, σ and τ about $\cdot 9\mu$ just beyond the red end of the spectrum; Φ, Ψ and Ω between 1μ and 2μ farther

[1] 'Researches on Solar Heat,' *Professional Papers of the Signal Service*, No. xv, Washington, 1884.

in the infra-red, X at $2{\cdot}5\,\mu$ to $2{\cdot}8\,\mu$ and Y and Z, still farther in, where the original intensity is very small.

Long-wave radiation which is emitted by black bodies has a notably transparent band from $9\,\mu$ to $11\,\mu$; great absorption by water-vapour from $6\,\mu$ to $8\,\mu$ and from $13\,\mu$ to $20\,\mu$, with moderate absorption elsewhere; there is also absorption by ozone about $10\,\mu$ and strong absorption by carbon dioxide from $12\,\mu$ to $18\,\mu$. So that on the average only 10 per cent. of "black-body-radiation" gets through the atmosphere as radiation, the remaining 90 per cent. being absorbed and transformed into heat. The actual percentage absorption, however, depends like many other things upon the weather.

Absorption

The absorption of energy by a transparent medium is of considerable interest as illustrating once more the rule of logarithmic relationship between two quantities which later we shall see expressed in the vertical distribution of pressure and in other ways. The law is originally due to Bouguer and can be expressed thus:

As the radiation passes through successive layers of equal thickness of a medium which is only imperfectly transparent on account of absorption, each successive layer absorbs *the same fraction of the energy of the radiation which impinges upon its front surface.* Imagine for example radiation passing through successive centimetres of water. If the first centimetre absorbs one-tenth of the incident radiation the next will absorb one-tenth of the remainder and the next one-tenth of what has passed the second and so on. Hence the first layer will have transmitted $\frac{9}{10}$, the second $(\frac{9}{10})^2$, the third $(\frac{9}{10})^3$ and the nth layer $(\frac{9}{10})^n$. The amount absorbed will be the difference between the original I_0 and $(\frac{9}{10})^n I_0$. The amount which survives to pass the nth layer is $I_0\,(\frac{9}{10})^n$.

The relation may be expressed in the form $I = I_0 a^n$ where a represents the fraction of the incident radiation transmitted through unit thickness of the absorbing substance and is therefore defined as the coefficient of transmission.

This is a logarithmic or exponential law which may be expressed as

$$\log I - \log I_0 = n \log a, \text{ or } I = I_0\, e^{-kn}, \text{ where } e^{-k} = a.$$

In actual practice the effect of the selective absorption, that is the difference in the effect of the medium upon waves of different wave-lengths, is of great importance, and prevents the application of any simple formula such as that quoted above. This important principle of the dependence of absorption upon wave-length is most effectively illustrated in the case of light when wave-length is identified as colour. Thus we may have a glass like the cobalt blue which is moderately transparent for blue light over a considerable range of wave-lengths, and much more perfectly transparent for a narrow band in the deep red. Let us suppose that the transparency or coefficient of transmission for the two kinds of light is $\cdot 1$ for blue and $\cdot 9$ for red for a layer of 1 mm, then for two millimetres the transmitted blue will be $\cdot 01$ and for the transmitted red $\cdot 81$, for a third layer $\cdot 001$ for the blue and $\cdot 729$ for the red. Hence a layer

a few millimetres in thickness is practically no longer blue glass but very definitely red glass. The experiment can easily be tried with plates of cobalt blue glass and the same kind of thing can be illustrated in atmospheric absorption.

The absorption of the atmosphere will depend upon the length of path in accordance with the formula $I = I_0 a^n$. The fraction a is generally different for different wave-lengths. It is by the application of this principle that Langley and other observers following him arrive at a satisfactory limiting value for the energy of solar radiation outside the atmosphere.

By choosing the time of suitable solar altitude the length of path can be obtained which is twice that of another given altitude, and in this way the limiting value can be obtained by a species of extrapolation.

Selective absorption of solar radiation

Item C of the balance-sheet

For every separate wave-length in the spectrum of a radiating body there is an appropriate coefficient a in the formula $I = I_0 a^n$; but in practice the receiving part of any instrument for measuring the energy will cover a range of wave-lengths depending on the degree of separation or dispersion in the spectrum under examination; hence the experimental values of the coefficient represent the absorption for a certain range of wave-length rather than that for a single point in the spectrum. The coefficient of Bouguer's formula thus obtained cannot be applied to spectra which have a curve of distribution materially different from that for which the coefficients were obtained.

For the spectral distribution represented in fig. 72 the principal absorption bands and the materials which are regarded as responsible for the absorption are placed as follows[1]:

Principal absorption bands of the solar spectrum

B	0·69 μ	Oxygen	Φ	1·13 μ	Water-vapour
a	0·72	Water-vapour	Ψ	1·42	,, ,,
A	0·76	Oxygen	Ω	1·89	,, ,,
—	0·81	Water-vapour	ω_1	2·01	*
$\rho\sigma\tau$	0·93	,, ,,	ω_2	2·05	*

* According to Hettner[2], ω_1 extends from 1·91 to 1·97 μ, ω_2 from 1·97 to 2·03 μ; both are accounted as due to water-vapour.

The absorption of water-vapour is of fundamental importance in the section of long-wave radiation, but it has little effect on the short-wave radiation from the sun.

> At sea-level on a clear day when the sun is in the zenith only about 6 to 8 per cent. is absorbed from the direct solar beam within the great infra-red bands...in its passage to the surface of the earth.
>
> (F. E. Fowle, *Annals of the Astrophysical Observatory*, vol. IV, p. 274; 'Water-vapour transparency to low-temperature radiation,' *Smiths. Misc. Coll.* vol. LXVIII, 1917.)

[1] F. E. Fowle, 'The transparency of aqueous vapour,' *Astrophysical Journal*, vol. XLII, 1915, p. 400.

[2] *Ann. der Physik*, Vierte Folge, Bd. LV, Leipzig, 1918, p. 493.

The effect of the atmosphere is such that the distribution of the energy is changed as shown in fig. 73.

For different air-masses the figure shows the normal distribution among the respective wave-lengths, from ·35 μ to 2·35 μ, of the transmitted solar energy for Washington, 127 m: (I) Outside the atmosphere, air-mass 0, with maximum at ·48 μ; (II) with solar altitude 65° (air-mass 1·1), maximum at ·5 μ; (III) with solar altitude 30° (air-mass 2), maximum at ·68 μ; (IV) solar altitude

DISTRIBUTION OF THE ENERGY OF A SUNBEAM ACCORDING TO WAVE-LENGTH

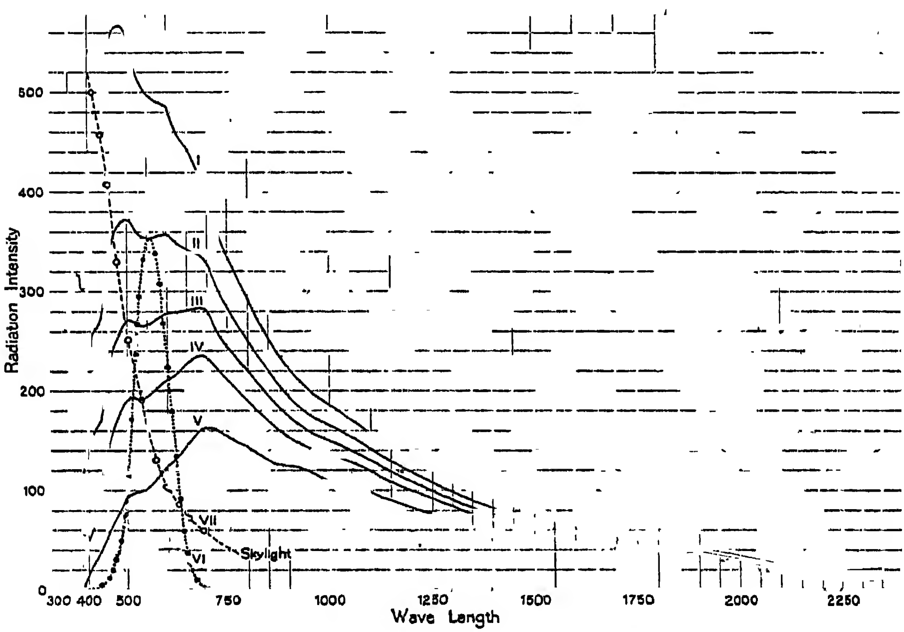


Fig. 73. The distribution of energy according to wave-length in a sunbeam as computed for the confines of the atmosphere (Curve I) and after traversing air-masses 1·1, 2, 3, 5 (Curves II, III, IV, V).

The scale of wave-lengths in millionths of a millimetre is set out along the base. The intensity of radiation for a given interval of wave-length is given by the ordinate according to a scale at the side.

Curve VI gives the relative brightness of the parts of the spectrum and Curve VII the intensity of "sky-radiation" for different wave-lengths at Mount Wilson.

(H. H. Kimball, *Monthly Weather Review*, vol. LII, 1924, p. 474.)

19·3° (air-mass 3), maximum at the same; and (V) solar altitude 11·3° (air-mass 5), maximum at ·7 μ. The additional curves in the diagram are (VI) the visibility of the radiation in respect of wave-length, and (VII) the intensity of the sky-radiation at Mount Wilson, 1730 m.

Dorno remarks that outside the limit of the atmosphere the energy of solar radiation is made up of 5 per cent. of ultra-violet, 52 per cent. visible and 43 per cent. infra-red, whereas at the earth's surface (at Davos) for mean solar altitude the composition is less than 1 per cent. ultra-violet, 40 per cent. of

visible and 60 per cent. infra-red, thus showing the effect of the atmosphere in reducing the intensity of the radiation of smaller wave-lengths.

Oxygen, ozone and carbon dioxide. Other constituents of the atmosphere which absorb short-wave radiation are oxygen and ozone. The absorption by oxygen is small, being represented in Langley's curve (fig. 72) by the dips A and B. The absorption by ozone is extremely vigorous but it occurs only in the violet and ultra-violet part of the spectrum where the whole energy is very small, less than 1 per cent. of the whole spectrum.

The curve of energy of the solar spectrum shows very little intensity beyond the wave-length $\cdot 4\,\mu$ which marks the boundary of the visible spectrum on the side of short wave-lengths, and yet that part of the solar radiation is recognised at health-centres and elsewhere as an important climatic element. The wave-length $\cdot 3\,\mu$ is an important position, its behaviour in respect of ozone has attracted much attention. The solar spectrum appears to be cut off from $\cdot 3\,\mu$ downwards by air or by certain of its ordinary constituents, and the absorption is operative even at 9000 metres.

There is also an absorption band due to ozone at $\cdot 6\,\mu$ but its coefficient of absorption is small[1].

Carbon dioxide is an important constituent of the atmosphere for long-wave radiation, but it has practically no effect upon short waves.

Scattering or diffuse reflexion

We have already explained in our treatment of the blue colour of the sky that when a beam of sunlight passes through the atmosphere part of the energy is diverted from the regular rectilinear progression of the beam to a dispersal in all directions by scattering, which takes place from any material to be found in the way of the travelling radiation. The fraction of the energy which is scattered is inversely proportional to the fourth power of the wave-length, but any result of this differentiation of wave-lengths for different colours is only apparent when the wave-lengths are so small that the difference of wave-length makes a sensible difference in the fraction of the energy scattered.

When the particles are as large as those of a sandstorm or a cloud, waves of all lengths are similarly treated and we pass from scattering to diffuse reflexion from drops or particles of visible size.

The actual process appears to be applicable continuously to particles of larger and larger size through the irregular and diffuse reflexion from a cloud of water-drops or dust to the regular reflexion from a transparent plane surface with its polarising angle.

Much attention has been given to the question of diffuse reflexion by the workers of the Smithsonian Institution. We quote Fowle's summary[2] of the results:

> The non-selective scattering of energy varies continuously with the wave-length and is easily expressed as a continuous function of the wave-length. In the case of

[1] Ch. Fabry, Conseil International de Recherches, *Deuxième Rapport de la Commission des relations entre les phénomènes solaires et terrestres*, Paris, 1929, p. 49.

[2] *Astrophysical Journal*, vol. XLII, 1915, p. 394.

the permanent gases of the atmosphere above Mount Wilson on clear days the scattering is almost purely molecular and may be computed from the number of molecules present in the path. In the case of water-vapour the losses are considerably greater than would be expected from purely molecular scattering and are apparently caused by grosser particles associated with water-vapour. The scattering varies so slowly with the wave-length that the coefficients which express it depend but slightly upon the purity of the spectrum....

Above an altitude of 1000 metres dust is generally negligible on clear days. At sea-level the dust coefficients are very variable from day to day. They are probably nearly the same for all wave-lengths less than 3 μ. The average scattering caused by the dust above Washington on clear days is about 9 per cent. On one of the clearest days on which observations have been made there it amounted to 3 per cent. (February 15, 1907).

The atmospheric losses from the incoming solar energy comprise five parts:

(1) that due to the general scattering by the molecules of the permanent gases of the atmosphere;

(2) that due to the general scattering associated with water-vapour;

(3) that due to selective (banded) absorption of the permanent gases of the atmosphere;

(4) that due to the selective (banded) absorption of water-vapour;

(5) that due to dust.

For the average amount of water-vapour at Mount Wilson (0·7 cm precipitable water) the losses of solar energy due to dry air, the water-vapour, and both together are on the average when the sun is in the zenith 0·15 cal, 0·17 cal, 0·32 cal; and for sun at altitude 20°, 0·39 cal, 0·25 cal, 0·64 cal.

For Washington on the driest day (0·5 cm precipitable water) the corresponding values are: for zenith sun, 0·19 cal, 0·19 cal, 0·38 cal; for sun at 20°, 0·44 cal, 0·37 cal, 0·81 cal.

(The loss due to dust at Washington is included with that due to water-vapour.)

On the average about half the loss of energy in coming through the atmosphere is due to the permanent gases and half to water-vapour.

Tables and other particulars are given in the memoir to which reference has been made.

The light which is diffused by regular or irregular reflexion from earth or cloud is included under the term albedo.

Molecular scattering and atmospheric absorption

The scattering from the molecules of the air in the regions beyond the clouds is regarded as being in accordance with the formula of Rayleigh's theory of scattering[1]. It is generally accepted as an explanation of the blue of the sky.

In a note to *Nature* in 1909, Sir Arthur Schuster cited Lord Rayleigh's formula for the scattering of light by small particles and drew the conclusion that if k were the coefficient of extinction of energy, μ the refractive index,

[1] Rayleigh, *Scientific Papers*, vol. 1, Cambridge, 1899, p. 92; *see also* L. V. King, 'On the scattering and absorption of light in gaseous media with applications to the intensity of sky-radiation,' *Phil. Trans.* A, vol. CCXII, 1913, p. 375. The intensity of the light scattered from a cloud is thus equal to $\frac{A^2(D'-D)^2}{D^2}\sin^2\alpha\,\frac{\pi^2\Sigma T^2}{\lambda^4 r^2}$, where T is the volume of the disturbing particle, r the distance of the point (illuminated by the scattered light) from it, λ the wave-length, D and D' the original and altered densities of the medium.

n the number of particles per cc ($2{\cdot}72 \times 10^{19}$), $k = 32\pi^3 (\mu - 1)^2/3n$, and if H is the height of the homogeneous atmosphere above the point of observation and no light is lost in any other way than by molecular scattering, the fraction of light which would reach the observer would be e^{-kH}.

Calculating the loss by molecular scattering in this way for different wave-lengths he placed the figures for Washington and Mount Wilson in juxta-position with the actual loss of energy computed from observations of radiant energy for an exceptionally clear day, 15 February 1907 for Washington and 11 October 1906 for Mount Wilson.

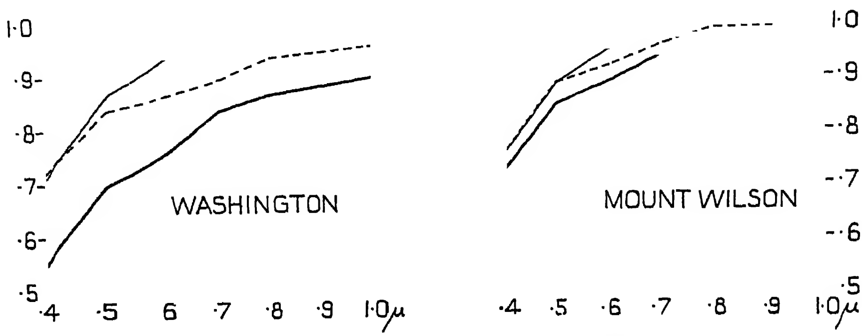


Fig. 74. Fractional loss of energy of solar radiation of different wave-lengths in the visible spectrum as computed from the scattering of molecules of air (full thin line) and as observed on exceptionally clear days (- - -) and on all days (full thick line) at Washington and Mount Wilson.

The comparison is represented in fig. 74 in which the thin full line shows the relation to wave-length of the loss by molecular scattering as calculated from the formula, the broken line the loss computed from the observations on the selected clear days, and the thick full line that computed for the observations of all days. It is evident that on clear days at Mount Wilson the loss is accounted for by molecular scattering.

The human body as radiator

The short waves of solar radiation include those which are regarded as peculiarly active in their effect upon the human organism, ultra-violet rays especially; but on that subject we have still much to learn. With practice the human body displays its capacity for reacting to the radiative conditions of its environment. The phosphorescence of living tissue in invisible ultra-violet is an example. Dr Leonard Hill with his "kata-thermometer" assimilates us to wet bodies. We might use another analogy. Long experience goes to show that where short waves are plentiful and powerful humanity expresses itself as a "black body," a good absorber and a good radiator; but where short waves are scanty it becomes a white body with the albedo that belongs thereto, and perhaps helps to protect it against the loss of radiation by long waves to which we now turn our attention.

Long waves

Having set out the conditions of transmission and absorption of short-wave radiation of the solar spectrum, we proceed now to consider the conditions which apply in the case of long-wave radiation such as that which is emitted by a black body at ordinary terrestrial temperatures between 180tt and 320tt. We may regard the radiation at 287tt, represented in fig. 49, as typical.

With a rock-salt prism long waves may be appreciated also in the solar spectrum and have actually been measured between 9 μ and 13 μ, but the scale of intensity is of the order of one-tenth per cent. in comparison with the radiation between 2 μ and 3 μ. We shall therefore, in what follows, disregard the sun as a long-wave radiator.

Water-vapour. The constituents of the earth's atmosphere which we have specially to consider in relation to the transmission and absorption of long waves are again water-vapour, carbon dioxide and ozone. Of these water-vapour is by far the most important. And again we may regard the influence of water-vapour upon the radiation in respect either of emission or absorption as depending upon the total quantity of water involved. On that account the quantity of water-vapour in a layer of the atmosphere is expressed in the phraseology of the Smithsonian Institution as the thickness of precipitable water, or the depth of water equivalent to the vapour. With this understanding Fowle gives the following table of intensity of radiation from a black body at 287tt between certain limits of wave-length and the absorption of the atmosphere with certain thickness of precipitable water, wherewith any effect of absorption by carbon dioxide 4 μ to 6 μ and 13 μ to 16 μ, and that of "a band of unknown origin at about 10 μ," would be included. Ozone has also absorption bands between 5 μ and 10 μ.

Atmospheric absorption of earth-radiation

Wave-length	Energy of black body at 287tt	Percentage absorption for precipitable water ·003 cm	·03 cm	·3 cm	3 cm
3 to 4 μ	5	10	30	50	75
4 to 5	50	15	45	70	95
5 to 6	142	16	43	66	95
6 to 7	242	45	85	95	100
7 to 8	315	13	42	85	100
8 to 9	360	0	2	40	50
9 to 10	380	0	0	0	15
10 to 11	370	0	2	5	40
11 to 12	350	0	0	4	10
12 to 13	320	0	0	13	20
13 to 16	810	100	100	100	100
16 to 20	510	(90)	100	100	100
20 to 30	900	?(70)	?(80)	?(90)	100
30 to 40	300	?(100)	?(100)	?(100)	100
40 to 50	150	(100)	(100)	(100)	(100)
50 to 60	75	(100)	(100)	(100)	(100)
3 to 60	5279 [39 kw/(10 m)^2]	49	57	66	75

In accordance with the values given in the last line of the table the vertical transmissions of the earth's radiation are therefore 51, 43, 34, and 25 per cent., corresponding to 0·003, 0·03, 0·3 and 3 cm precipitable water. Further, applying these last figures to the transmission of radiation outwards in all directions from a horizontal surface at sea-level, assuming Lambert's cosine law, and 1 cm precipitable water, it is found that 28 per cent. of the earth's radiation under such circumstances passes directly out to space.

Considering the rate of growth of percentage absorption with increasing atmospheric humidity, and that

(1) ...average precipitable water is to be regarded as 3 cm;

(2) outgoing terrestrial radiation is emitted at such angles of emergence that the air-mass of average emergence is 1·8 times that of zenith transmission;

(3) that the absorption of ozone in the high atmosphere seems to cut off one-fifth of the surface terrestrial radiation transmitted by the lower atmospheric layers,

we conclude that of the earth's total surface-emission it is unlikely that more than 20 per cent. is transmitted by the atmosphere to space in fair weather. Allowing for total absorption 50 per cent. of the time by clouds, the final result is 10 per cent. as the transmission to space from the earth's surface.

(*Annals of the Astrophysical Observatory*, vol. IV, 1922, p. 286.)

The table makes no allowance for the variation in the absorption by the same quantity of water-vapour under different conditions of total pressure, which for the water-vapour band about 2·7 μ was shown by Frl. v. Bahr[1] to range from 4·6 per cent. to 12 per cent. for a variation of total pressure from 140 mb to 1000 mb. For the atmosphere, in which the greater part of the vapour is in the lower layers, the correction is not of great magnitude. We are not yet in a position to utilise it quantitatively in meteorological practice.

In the application of the facts of radiation to account for the thermal structure of the atmosphere it has been the practice to regard water-vapour as a "grey" body, that is one which absorbs all wave-lengths in the same proportion. In a memoir recently presented to the Royal Meteorological Society, Dr Simpson[2] has pointed out that that assumption leads to erroneous conclusions, or at least that the agreement with observation of a conclusion based upon it may be regarded as coincidence.

Further light has been thrown on the selective absorption and corresponding emission of long-wave radiation by the atmosphere through the recent investigations of G. Hettner[3], who has determined experimentally the absorption for layers of steam of 109 cm to 32·4 cm at relatively high temperatures. The results are expressed in a diagram representing the coefficients of absorption for wave-lengths between 0·8 μ and 36 μ. In this diagram absorption is shown as complete for wave-lengths between 2·5 μ and 2·9 μ, between 5·5 μ and 8 μ and from 14 μ onwards to the extent of the range of wave-length investigated. The maxima agree fairly well with Abbot and Fowle's results upon which the diagram of fig. 49 is based.

[1] Quoted by Fowle, *Smiths. Misc. Coll.* vol. LXVIII, No. 8, 1917, p. 7.

[2] 'Further studies in terrestrial radiation,' *Memoirs of the Royal Meteorological Society*, vol. III, No. 21, 1928.

[3] *loc. cit.*, *vide* p. 148.

From Hettner's diagram Dr Simpson has prepared a curve of selective absorption for ·3 mm of precipitable water-vapour with which he has incorporated the coefficients for CO_2. The diagram is reproduced in fig. 75.

SELECTIVE ABSORPTION OF LONG-WAVE RADIATION

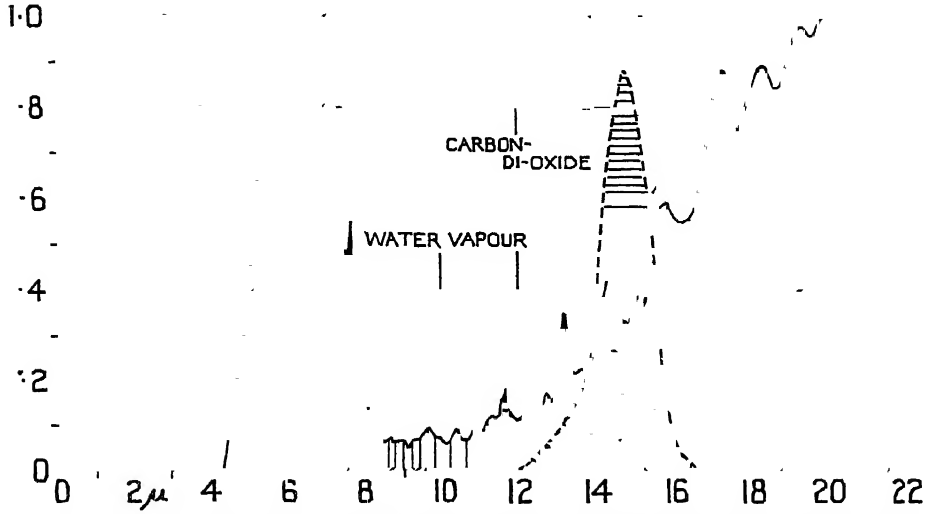


Fig. 75. Selective absorption of long-wave radiation by water-vapour (vertical ruling) calculated as the effect of vapour equivalent to ·03 cm of precipitable water and as the effect of carbon dioxide (horizontal ruling) calculated as ·06 g/cm² of carbon dioxide.

From that curve and corresponding curves of emission based upon it he has computed the outgoing radiation from the earth which works out remarkably near to the value for the incoming radiation computed from the solar constant with an allowance of 43 per cent. for the earth's albedo. The diagram makes no allowance for absorption by dust; the coefficients employed were derived from experiments on water-vapour in closed tubes, the conditions for which are not fully realised in the open air.

In a subsequent memoir Dr Simpson has given the computation of the energy received and lost by different parts of the earth's surface in January and July. He finds that, broadly speaking, each hemisphere in its own summer is receiving radiation in excess of the loss to space, while in each month of the year the total received by the whole earth is in agreement within 2 per cent. with the energy lost.

Other conclusions derived from the same premises we shall refer to in a subsequent chapter.

Carbon dioxide. Of the gaseous constituents of the atmosphere other than water-vapour, carbon dioxide is the only one which need be considered. Apart from ozone, oxygen and nitrogen are said not to be absorbing agents.

In atmospheric conditions the absorption of carbonic-acid gas in the spectrum of the earth appears to be confined to two bands extending from wave-length 3·6 μ to

5·4 μ and from 13·0 μ to 16·0 μ respectively. In these bands its absorption is nearly total from 4·0 μ to 4·8 μ and from 14·0 μ to 15·6 μ even when carbonic acid is present in much less quantities than the atmosphere contains. But the areas included by the energy curve of the "black body" at 287·2tt from 3·6 μ to 5·4 μ and from 13·0 μ to 16·0 μ are 0·5 per cent. and 13·5 per cent. of the total area of the curve... in the absence of water-vapour the total absorption possible by carbonic-acid gas would be 14 per cent. (*Annals of the Astrophysical Observatory*, vol. II, 1908, p. 172.)

Schaefer... concludes that a variation in the amount of CO_2 in the atmosphere would not materially affect its absorbing power for solar radiation, since the amount present, equivalent for vertical transmission to a path of 250 cm at a pressure of 760 mm, is much more than sufficient to exert full absorption for the width of band corresponding to the density of CO_2 in the atmosphere.

. . . .

If we use the results obtained, we find that for CO_2 at a pressure less than 760 mm, the maximum possible absorption for radiation from a 15° C [288tt] source in the bands discovered is about 18 per cent. of the complete energy in the spectrum of a perfect emitter, as given by Planck's formula.

(E. Gold, 'The isothermal layer of the atmosphere and atmospheric radiation,' *Proc. Roy. Soc.* A, vol. LXXXII, London, 1909, pp. 49, 51.)

It is the absorption of long-wave radiation by an atmosphere which contains carbonic acid and the absence of any corresponding absorption of the short waves coming from the sun which has suggested to Arrhenius, Ekholm and others the idea of a possible change in the climate of the world by the "greenhouse effect" of an increased amount of carbon dioxide in the atmosphere. (See W. J. Humphreys, *Physics of the Air*.)

Water. The reference of the radiation of water-vapour to the equivalent thickness of liquid water points to the necessity for citing what information we possess about the absorption of radiation by water in the ordinary liquid form. That water, like window-glass, is remarkably opaque to long-wave radiation is common knowledge in the physical laboratory where a glass cell containing water, and still better a solution of alum, is used to filter out the "heat rays" of a projection lantern.

According to Fowle[1], W. Schmidt, using data of Aschkinass, computes the absorption by various thicknesses of water as follows:

1 mm	absorbs everything for wave-lengths greater than 2·4 μ
1 cm	absorbs everything for wave-lengths greater than 1·5 μ
10 cm	absorbs everything for wave-lengths greater than 1·2 μ
10 m	absorbs everything for wave-lengths greater than 0·9 μ
100 m	absorbs everything for wave-lengths greater than 0·6 μ

The wave-length 0·6 brings us within the visible part of the spectrum for the transmission of which through water we have already given the interesting results of Knudsen (vol. II, p. 51) for wave-lengths from ·65 μ to ·40 μ.

Here we may supplement Knudsen's figures by results due to Nichols[2] for wave-lengths outside the range of the visible spectrum.

[1] *Smithsonian Miscellaneous Collections*, vol. LXVIII, No. 8, 1917, p. 49, and 'Absorption der Sonnenstrahlung in Wasser' by W. Schmidt, *Met. Zs.* Bd. XXV, 1908, p. 321.
[2] Quoted by Fowle, *loc. cit. supra*.

Percentage transmission of 1 *cm liquid water*

Wave-length λ	·779	·865	·945	1·19	1·41 to 2·8 μ
Percentage transmission	76·2	74·4	58·4	14·4	too small to measure

Water in a thickness of 2 cm is practically completely absorbing for rays emitted by a body at temperatures 373tt and lower, and would be a perfect radiator at such temperatures, provided its reflecting power for these rays were zero.

For angles of incidence greater than 75°, reflection is above one-third total, for vertical incidence the reflection is about 2 per cent. for radiation of a black tea-kettle at 373tt.

We conclude that water lacks but 4·4 per cent. of radiating and absorbing as much as the perfect radiator.

For angles of incidence greater than 75° water is highly reflecting. For angles of incidence up to 15° water at temperatures under 303tt radiates 99·5 per cent. as much as the perfect radiator.

(Based on *Annals of the Astrophysical Observatory*, vol. IV, 1922, p. 290.)

The similarity of water and glass as absorbers of long-wave radiation may be emphasised by the fact that a glass screen is often used to keep off the radiation of a fire while it allows the light to pass through. In like manner the glass of a greenhouse offers practically no obstruction to the passage of the solar energy, but is impervious to the long-wave radiation from the plants in the interior. It produces, in fact, the same sort of effect as a cloud-cover in nature; it prevents the loss of heat on balance which is suffered by any body exposed to the clear sky.

Some figures which express the selective property of glass are given below; the figures represent the coefficient of transmission a in the Bouguer formula $I_x = I_0 a^x$.

	Unit of thickness = 10 cm Wave-length ·375 μ to ·677 μ		Unit of thickness = 1 cm ·7 μ to 2·9 μ or to 3·1 μ
Ordinary light flint	a ·388 increasing to ·939	Borate crown	1·00 diminishing to ·18
Ordinary silicate crown	a ·583 increasing to ·903	Crown	·99 diminishing to ·29

(*Smithsonian Physical Tables*, 7th edition, 1920, p. 302.)

The intensity of the energy reflected from a surface of water is not by any means independent of the wave-length. Short waves are easily reflected whereas the reflexion of long waves is very defective. We have at the present no figures on the subject.

* * *

We have now set out the information which we have been able to glean about the physical processes by which the earth becomes possessed of its supply of energy from the sun. The line of investigation has been in the direction of the differentiation of the energy actually received and the evaluation of the separate items, presumably with the object, so far as meteorology is concerned, of subsequently integrating the effects of the items collectively upon the atmosphere.

We have expressed the energy in terms of kilowatt-hours per square dekametre, an engineer's unit. The practice has been objected to on the ground that measures of radiation have been expressed in the past as their equivalent

in g cal/(cm^2 min) and should continue to be so expressed because the relation of radiant energy to heat-energy is so intimate.

But it is exactly the relation of the radiant energy to heat-energy that meteorologists require to explore. The question which appears in the foreground of the study of the atmosphere is: How much of the solar energy is really converted into gramme calories in the circumstances actually obtaining in the open air, and when? The implication of the question is obscured rather than illuminated if we say: "The whole of it, if and when it gets into the standard black body of the Smithsonian Institution." Natural circumstances are not those of a perfectly black enclosure. In the information which we have collected there is evidence of many a slip between the cup of the sun and the lip of the Smithsonian enclosure. Heat is a peculiar form of energy, as future chapters will explain.

LONG-WAVE RADIATION FROM EARTH AND AIR

Items E, F, G, K, M of the balance-sheet

The balance-sheet of p. 107 shows the energy obtained by short-wave radiation from the sun to be redistributed over space external to the earth's atmosphere partly by immediate reflexion represented by the albedo, and partly by long-wave radiation emanating from sea or earth and air. The energy which thus emanates depends upon the nature of the surface and on the temperature of the body from which it originates; and those of course are different for different localities and at different seasons; the direct measurement of the energy so emitted on selected occasions is obviously an important part of the study of radiation in relation to meteorology. Against the loss by radiation to the surrounding atmosphere must be set the amount of long-wave radiation of the atmosphere to the surface, depending upon the temperature of the air and the water-vapour which it contains. The difference between the outgoing and incoming radiation is often treated under the name of nocturnal radiation. Objection is taken to the name because the radiation from the surface is continuous day and night over the whole earth with appropriate degrees of intensity in different parts; and in fact it is its continuance throughout the 24 hours which enables it, in the long run, to balance the more potent influence of the sun's radiation which on the average has an innings of only 12 hours. On that account there is some advantage in calling it nocturnal because radiation in the absence of the sun is the ideal that we wish to have in mind; and we wish to consider it quite independently of the solar radiation which overpowers it for a large part of the day. Long-wave radiation is perhaps the best name.

It forms the subject of a memoir by Anders Ångström, 'A study of the radiation of the atmosphere,'[1] in which he recounts the results of special expeditions under very favourable conditions to Algeria and California in aid of which the upper air investigation of the U.S. Weather Bureau was invoked with good effect.

[1] *Smithsonian Miscellaneous Collections*, vol. LXV, No. 3, Washington, 1915.

Ångström was concerned to measure what he and others call the "effective radiation" from a black body exposed to the sky and thereby to obtain a measure of the counter-radiation of the atmosphere; any surface emits part of its own thermal energy by radiation on account of its temperature and receives in return the energy of radiation emanating from the sky. The effective radiation is the difference between these two and expresses the rate at which a body at the surface of the earth loses heat on account of radiation alone. The effective radiation is expressed in Dines's balance-sheet by the difference between G (long-wave radiation from the earth) and E plus M (long-wave radiation from the atmosphere).

In the previous section we have noted that Dr Simpson has made a successful computation of the outgoing radiation of the whole earth upon the basis of the absorption of radiation by water-vapour as represented in fig. 75. From that diagram he concludes that a layer of atmosphere which contains enough water-vapour to form ·3 mm of water will behave as a black body for radiation of wave-length 5·5 μ to 8 μ and from 14 μ onwards. On that basis the stratosphere becomes a notable radiator and absorber. For the radiation of the wave-lengths which suffer only partial absorption allowance is made by dividing the areas on the diagram representing black-body radiation for the appropriate temperature. We may without serious difference in the final result divide the wave-lengths instead of dividing the area and take atmospheric radiation from a layer with ·3 mm water as black-body radiation for the wave-lengths 5·5 μ to 8 μ and from 14 μ onwards, and perfect transparency for the rest, with however a certain allowance for carbon dioxide.

In this section we are to deal with actual observations of the balance of radiation between earth and sky. Reference may be made by the observers to the humidity of the atmosphere at the time of observation but no numerical estimate of air-radiation is made on the basis of the radiating power of water-vapour for different wave-lengths. The results which we are quoting may be read usefully in the light of the more recent representation of the basis of the radiating power of the atmosphere. But the suggestion entails more inquiry than we can follow up at present. We leave it to the reader, and proceed with our recital of the results of observations of long-wave radiation.

For the purposes of measurement a black body is chosen as the terrestrial radiator. The black body employed has taken different forms with different observers. J. Maurer, 1886, who was the first to measure effective radiation, used a circular copper disk; J. M. Pernter, 1888, "a similar method"; Homén, 1897, two equal copper plates alternately exposed and covered; Christiansen, metal disks placed on a water-surface and exposed to the sky, the thickness of ice formed on the disks was the index; F. Exner, 1903, the strips of a compensating pyrheliometer; K. Ångström, 1905, strips which are the sensitive part of the black body of a pyrgeometer; Lo Surdo, 1908, "the same type"; A. Ångström, 1912, also the same, to be mentioned later.

Ångström gives a table of the effective radiation from a square centimetre

of surface in gramme calories per minute from which we have taken the following:

Table of measures of effective radiation of a black surface exposed to clear sky at night

	Date	Observer	Place	Height	Mean effective radiation		Temperature
				m	g cal/cm² min	kw/(10 m)²	tt
1887	June 13–18	Maurer	Zürich	500	0·128	8·92	288–291
1888	Feb. 29	Pernter	Sonnblick	3095	0·201	14·0	265
1888	Feb. 29	Pernter	Rauris	900	0·151	10·5	—
1896	Aug.	Homén	Lojosee	—	0·17	11·8	—
1902		Exner	Sonnblick	3106	0·19	13·2	—
1902	July 1	Exner	Sonnblick	3106	0·268 (max.)	18·7	—
1904	May-Nov.	K. Ångström	Upsala	200	0·155	10·8	273–283
1908	Sept. 5–6	Lo Surdo	Naples	30	0·182	12·7	293–303
1912	July 10 to Sept. 10	A. Ångström	Algeria	1160	0·174	12·1	293

Radiation from a black body at 288tt according to Stefan's law with Kurlbaum's constant would be 0·526 g cal/cm² min, 36·7 kw/(10 m)².

Homén draws from his observations on the [long-wave] radiation between earth and sky the following conclusions: (1) if the sky is clear, there will always be a positive radiation from earth to sky, even in the middle of the day; (2) if the sky is cloudy, there will always be, in the daytime, a radiation from sky to earth; (3) in the night-time the radiation for a clear as well as for a cloudy sky has always the direction from earth to sky.

. . . .

Contrary to Homén [Lo Surdo] finds a positive access of radiation from the sky even when the sky is clear.

. . . .

In agreement with former investigations made by Maurer and Homén, Exner found the [effective] radiation to be relatively constant during the night...there are tendencies to a slight maximum...one or two hours before sunrise.

During a clear and especially favourable night [Lo Surdo] found a pronounced maximum about two hours before sunrise.

(A. Ångström, *A study of the radiation of the atmosphere*, Washington, 1915.)

Dines's general conclusions are recorded on pp. 163 to 167.

The pyrgeometer[1] which Anders Ångström used depends upon four thin manganin strips in one plane stretched across an aperture in a plane surface which forms the face of the instrument; two are black, two gilded, each carries a thermo-junction. Loss of heat from the black is compensated by an electric current and the compensation is indicated by the thermoelectric circuit, and the necessary current by an ammeter. A black hemisphere of known temperature is used to standardise the instrument.

Observations were made at Bassour in Algeria in 1912, the year when the dust of Katmai was operative, and in 1913 at a number of stations in California. At the same time upper-air observations were carried out by the staff

[1] 'Recording Nocturnal Radiation,' *Meddel. från Statens Meteorologisk-Hydrografiska Anstalt*, Bd. III, No. 12, Stockholm, 1927.

of the U.S. Weather Bureau by free balloons at Avalon, and by captive balloons at Lone Pine and Mount Whitney.

The observations of radiation were as follows:

	Station	Month	Number of: days	obs.	Altitude m	Range of effective radiation g cal/cm² min	kw/(10 m)²
1912	Bassour, Algeria	July–Sept.	38	—	1160	0·138 to 0·220	9·62 to 15·33
1913	Indio	July	3	28	0	0·118 to 0·193	8·23 to 13·45
1913	Lone Pine	Aug.	9	100	1140	0·127 to 0·241	8·85 to 16·80
1913	Lone Pine Canyon	Aug.	5	25	2500	0·147 to 0·226	10·25 to 15·75
1913	Mt San Antonio	July	2	23	3000	0·131 (clouds after 3 h) to 0·225	9·13 to 15·68
1913	Mt San Gorgonio	July	2	14	3500	0·198 to 0·223	13·8 to 15·54
1913	Mt Whitney	Aug.	9	83	4420	0·150 (after foggy afternoon) to 0·228	10·46 to 15·89
1913	Mt Wilson	Aug.	1	16	1730	0·140 to 0·155	9·76 to 10·80

Here we may supplement Ångström's results by the following observations published elsewhere.

With a clear sky the nocturnal long-wave radiation of the earth at Davos[1] amounts to:

Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	April	May	
0·182	0·178	0·183	0·180	0·205	0·216	0·185	0·183	g cal/cm² min
12·7	12·4	12·8	12·5	14·3	15·1	12·9	12·8	kw/(10 m)²

these being the mean values from twilight to twilight.

Long-wave radiation from the atmosphere

The measure of radiation from the atmosphere is the difference between the fourth power radiation of a black body at the temperature of observation and the observed effective radiation. The observations are utilised mainly to study the relation between the effective radiation and the corresponding temperature and humidity of the air at the time of observation, as a means of ascertaining the dependence of the radiation of the atmosphere upon those quantities. A specimen of the comparison in the case of observations at Mount Whitney on the nights on which upper-air observations were obtained with a captive balloon is given in fig. 82, chap. v.

The main results and conclusions as summarised by Ångström are as follow:

1. The variations of the total *long-wave radiation of the atmosphere* at low altitudes (less than 4500 m) are principally caused by variations in temperature and vapour-pressure.

2. The total radiation received from the atmosphere is very nearly proportional to the fourth power of the temperature at the place of observation. [$E = CT^a$; the weighted mean of a for Indio and Lone Pine is 4·03.]

[1] A. Ångström and C. Dorno, trans. in *M.W. Rev.* vol. XLIX, Washington, 1921, p. 135.

3. The radiation is dependent upon the [absolute] humidity. An increase in the vapour-pressure of the atmosphere will increase its radiation in a manner that has been expressed by an exponential formula. [$E = \cdot 439 - \cdot 158 \times 10^{-\cdot 069q}$ where q is the vapour-pressure presumably in millimetres.]

4. An increase in the vapour-pressure will cause a decrease in the effective radiation from the earth to every point of the sky. The fractional decrease is much greater for large zenith angles than for small ones.

5. The total radiation which would be received from a perfectly dry atmosphere with a temperature of 293tt at the place of observation would be about

$$0{\cdot}28 \text{ g cal/cm}^2 \text{ min}, \ 19{\cdot}5 \text{ kw/(10 m)}^2.$$

6. The radiation of the upper dry atmosphere would be about 50 per cent. of that of a black body at the temperature of the place of observation. [Mount Wilson.]

7. Any evidence in the observations of maxima or minima of atmospheric radiation during the night can be explained by the influence of temperature and humidity conditions.

8. There are indications that during the daytime the radiation is subject to the same laws that hold for radiation during the night time.

9. Increase of altitude of the station corresponds with a decrease or an increase in the effective radiation of a black body exposed to the sky according to the value of the lapse-rate of temperature, and the lapse-rate of vapour-pressure. At about 3000 m effective radiation generally has a maximum. An increase of the lapse-rate of humidity or decrease in that of temperature tends to shift the maximum to a greater altitude.

10. The effect of clouds is variable. Low and dense cloud banks cut down the effective radiation of a black body to about 0·015 g cal/cm² min, 1·05 kw/(10 m)². In the case of high and thin clouds the radiation is reduced by only 10 to 20 per cent. [from 0·28].

It is evident that, when the sky is cloudy, we can distinguish between three radiation sources for the atmospheric radiation: first, the radiation from the parts of the atmosphere below the clouds; secondly, the part of the radiation from the clouds themselves, which is able to pass through the inferior layer, and, in the third place, the radiation from the layers above the clouds, of which probably, for an entirely overcast sky, only a very small fraction is able to penetrate the cloud-sheet and the lower atmosphere.

Some measurements were taken in the case of an entirely overcast sky. In general the following classification seems to be supported by the observations:

	Average effective radiation g cal/cm² min	kw/(10 m)²
Clear sky	0·14 to 0·20	9·8 to 13·9
Sky entirely overcast by:		
Ci, Ci-st and St	0·08 to 0·16	5·6 to 11·2
A-cu and A-st	0·04 to 0·08	2·8 to 5·6
Cu and St-cu	0·01 to 0·04	0·7 to 2·8

11. The effect of haze upon the effective radiation is almost inappreciable when no clouds or real fog are formed. The great atmospheric disturbance in 1912 can only have reduced the effective radiation in Algeria by less than three per cent.

12. The probability is that radiation to the free air from large water-surfaces is nearly the same at different temperatures and consequently also in different latitudes. [Water is equivalent to 94 per cent. of a black body.]

Ångström's results are supported by additional observations by Sten Asklöf, *Geogr. Ann. Stockholm*, 1920, p. 253.

Perhaps the most important of Ångström's conclusions are first that a clear sky supplies radiation proportional to a power of the temperature at the place of observation not differing much from the fourth power; secondly that the

radiation from a clear sky above the surface of 4000 metres is about 50 per cent. of black-body (fourth power) radiation at the temperature of the station; and thirdly, that the radiation from large water-surfaces is very little different from that of a perfect radiator, perhaps 5 per cent. for radiation to the whole sky, and that in natural conditions when water-vapour tends to take a value corresponding with the temperature of the air the radiation from water is practically independent of temperature, and therefore also of latitude. The conclusion is important because so much of the radiation from the earth's surface which provides the item G in the balance-sheet must come from water. The behaviour of water is based upon Fresnel's formula for refraction, according to which the ratio of reflected light to incident light

$$= \frac{1}{2}\left\{\frac{\sin^2(i-k)}{\sin^2(i+k)} + \frac{\tan^2(i-k)}{\tan^2(i+k)}\right\},$$

where i is the angle of incidence and k the angle between the normal and the refracted beam.

Ångström relies to a considerable extent upon theoretical considerations of the radiation from layers of air of defined composition and temperature derived by what he cites as laws of radiation. It is not always easy to follow the application of theory to problems in nature in the absence of adequate knowledge of the constants which the formulae require.

Sky-searching for long waves

The question of the radiation from different parts of the sky has also been treated by W. H. Dines, but from a different angle and by a different method, to which we may now turn our attention. On p. 8 of vol. II we have already summarised his observations of radiation about sunset in the conclusion that, whatever be the time of year, on cloudless days near sunset at Benson in Oxfordshire the earth is losing heat at the rate of 7 kw/(10 m)^2. With the instrument described on p. 140 Dines explored the radiating capacity of the sky at Benson.

The results of the first year show remarkable fluctuations indicating a very variable sky; the effective temperature of the portion within the beam of his instrument in the course of daily observations during December 1920 ranged from 52° F to − 39° F (284tt to 233tt). "The highest temperatures which are always found with a dense layer of low clouds are in general a degree or two *below* that of the air at the time; the lowest temperatures which prevail on clear, but not necessarily on calm nights, are far below the corresponding minima in the screen or on the grass."

In treating the material of observations on radiation to clouds taken by myself and S. Asklöf, A. Defant has pointed out that the radiation in most cases is of exactly the amount which ought to be expected if the radiation took place against a black perfectly absorbing surface of the temperature at the altitude of the cloud-layer in question. In other words, the cloud-layer behaves very nearly like a perfectly absorbing surface as regards the long waves constituting the radiation from the earth.

(A. Ångström, 'Recording nocturnal radiation,' *Meddel. från Stat. Meteor.-Hydrog. Anst.* Bd. III, No. 12, Stockholm, 1927, p. 10.)

A. Ångström further expresses the corresponding fact as 50 per cent. of the radiation of a black body at the temperature of the surface, for the atmosphere above 4000 m. In Dines's notation and taking the temperature of the surface as 273tt, this would be that the equivalent black-body temperature for the sky is 230tt, a difference of 43tt or 77° F.

The variation in the equivalent black-body temperature of the sky which is here the subject of investigation might furnish a valuable addition to the usual meteorological indications of weather by registering variations in the water-vapour of the lower layers. Cases arise—an example will be given in chapter v—in which the main body of the atmosphere is sistible as dry air but as saturated air it is not; such conditions almost necessarily precede thunderstorms. The loading of the air with water-vapour, acting as a kind of trigger to set the upper air in motion, would be associated with increased long-wave radiation from the sky and consequently increased equivalent black-body temperature as determined by the sky-searcher. The condition preceding a thunderstorm is commonly indicated by calling the weather "close," and closeness in this connexion may be interpreted as an oppressive amount of counter-radiation of long waves from the sky which is in direct contrast with the exhilaration of a clear sky and its radiative possibilities.

W. H. Dines[1] gives the following correlation between T the temperature in the screen at the surface, V the vapour-pressure and S the long-wave radiation from the sky when cloudless (excluding diffuse solar, short-wave, radiation) on the supposition that the radiation from all parts is the same as from the zenith.

Correlation between S and V, ·80; between T and S, ·94; between V and T, ·77.

The observations on which the coefficients are based were made between February and August at hours from 7 h to 22 h with a decided preponderance of observations at 13 h and 18 h.

Dines deduces the equation

$$S = 330 + 4{\cdot}9\, v + 8{\cdot}7\, t,$$

where S is the radiation in g cal per square centimetre per day, v is the vapour-pressure in mb and t is the temperature in °C. The equation holds between temperatures 32° F (273tt) and 80° F (300tt).

We have given the result in the terms and symbols which Dines himself employed; our stock of units and symbols is really not sufficient to supply the requirements. Without risk of confusion, having regard to the economy which we have prescribed for ourselves in these matters, we have transformed Dines's equation to comply with our own rules and thus obtain

$$N\!\sim\; = 101{\cdot}2\mathrm{q} + 177{\cdot}8\mathrm{tt} - 4220,$$

where $N\!\sim$ is the long-wave radiation in kilowatts per square dekametre, q is the vapour-pressure in millibars and tt the temperature on the tercentesimal scale.

[1] *Q.J. Roy. Meteor. Soc.* vol. XLVII, 1921, p. 260.